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L. J. Conant Esq.
with the Respects
of the Author.

GEOMETRY:

APPLIED TO THE

MENSURATION

OF

LINES, SURFACES, SOLIDS,

HEIGHTS AND DISTANCES.

BY B. FRANKLIN CALLENDER,
Late Master of the Mathematical Department, Wells School, Boston.

"In His hand,
He took the golden compasses,
Prepared to circumscribe this universe
And all created things:
One foot he centred, and the other turn'd
Round through the vast profundity obscure;
And said, 'Thus far extend, thus far thy bounds,
This be thy just circumference, O! World.'"—MILTON.

NEW-YORK:
CHARLES S. FRANCIS—252 Broadway.
1836.

Math 8108.36
~~VI. 3459~~

1886, Mar. 19,
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PREFACE.

A FEW years ago, the author of the following work instructed a class of young men in Geometry and Mensuration. At that time, no suitable text book could be found of a sufficiently practical character to meet the wants of either the instructer or pupils in this branch of mathematics; and the only alternative was to use the best which could be procured, and supply the deficiency by preparing a written manuscript. That manuscript forms the foundation of the present treatise. Contributions have, from time to time, been made to the work, till the last year, during which it has received the almost exclusive attention of its author. At the urgent solicitation of several gentlemen who have examined the work, it is now presented to the public, with the belief that it will supply a want which has long been experienced; the want of a work containing a familiar illustration of the principles of Geometry, with a practical application of them to the Mensuration of lines, surfaces, and solids; and the general adaptation of the whole to the common purposes and business of life. Such has been the design of the author in the following work. How far that object has been accomplished, those for whom it has been prepared must decide. Simplicity in arrangement, perspicuity in illustration, and familiarity in the use of terms, are points which have been kept constantly in view.

In the following treatise, the principles are presented in an improved and intelligible form ; and some of them have never before appeared in any similar work. The Rules for finding the *sum*, the *difference*, and the *product* of the squares of numbers, without finding their squares — The Rule for finding the solidity of a wedge — The Rules for finding the chord of an arc of a circle, and for the construction of an ellipsis, are among the latter. The Rule for the Mensuration of loads of wood for fuel, is brief, simple, and easily applied. In Duodecimals, or multiplication of feet, inches, and parts, it is believed that an essential improvement has been made ; rendering that difficult, and, to some, almost inexplicable operation, plain and intelligible.

In Common and Decimal Fractions the operations are clear and perspicuous, and may be understood and applied as easily as similar operations upon whole numbers. These, with the Extraction of the Square and Cube Roots, form an important and indispensable introduction to Mensuration, Surveying, Navigation, and other practical branches of mathematics, and are recommended to the attentive examination of the student.

There are upwards of seven hundred practical questions and illustrations, relating to the business of the Architect Surveyor, Carpenter, Mason, Painter, Glazier, and other branches of mechanical operation. To the Agricultural, Mercantile, and Commercial community, this work will afford much useful information, applicable to their respective branches of labor and enterprise.

The sublime truths of mathematics, emanating, as they do, immediately from the "Great Geometer" of the Universe, and incapable either of improvement or alteration by any human mind, should not be concealed within literary palaces, nor obscured by the mist of technical phraseology when permitted to go out. They are the common property of all, and should be diffused as freely as the vital air, and the light of heaven ; that all may comprehend, admire, and enjoy them.

In a society, such as the American public, distinguished for the general diffusion of knowledge, every attempt to radiate and extend the light of science will be duly appreciated ; and he who dispels the darkness of ignorance from a single human mind, and supplies it with the light of truth, will not be unrewarded.

B. F. C.

JULY 1, 1836.

Andover, December 10, 1836.

Mr. Callender,

Dear Sir, — I have carefully examined your Geometry, and do most cheerfully bear testimony to its very great worth. It seems to have been sedulously prepared, and must prove a useful and convenient manual for schools and practical men. It comprises a far greater amount of knowledge than any similar work. Every part of it is perfectly clear and intelligible; and I cannot but urge teachers to examine it, that they may become acquainted with its merits.

Yours, CHARLES H. ALLEN.

[From the American Annals of Education, for January, 1837.]

This work appears to be at once simple, perspicuous, and judicious. We have seen no treatise in this department better adapted to the wants of teachers and pupils. The style of its execution is equally commendable.

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MATHEMATICAL NOTATION.

$=$	Equal, ...	$25=15+10$
$+$	Addition, ...	$15+10=25$
$-$	Subtraction, ...	$15-10=5$
$\times$	Multiplication, ...	$15\times 10=150$
$\div$	Division, ...	$15\div 10=1\frac{1}{2}$
^{2}	Square, ...	$5^2=25$
^{3}	Cube, ...	$5^3=125$
$\sqrt{}$	Square Root, ...	$100\sqrt{}=10$
$\sqrt[3]{}$	Cube Root, ...	$1728\sqrt[3]{}=12$

ERRATA.

Page 186 — Question 7 — Ans. 5.5632. The note should be, — "*and altitude*" *equal to a length of a side of the pyramid.*

GEOMETRY AND MENSURATION.

PART FIRST.

INTRODUCTION.

CHAPTER I.

FRACTIONS.

SECTION I.

Definitions.

Fractions signify parts of a thing broken, or of a number divided.

When a thing or number is divided into *two* equal parts, the parts are called *halves*. If a thing or number be divided into *three* equal parts, the parts are called *thirds*. If divided into *four* equal parts, the parts are called *fourths*. If divided into *five* equal parts, the parts are called *fifths*, &c. always receiving a name from the number of parts into which the thing or number has been divided.

When fractions are expressed in figures, two numbers are used, with a short line between them, as $\frac{1}{2}$, one half; $\frac{1}{3}$, one third; $\frac{2}{3}$, two thirds; $\frac{5}{7}$, five sevenths; $\frac{11}{12}$, eleven twelfths; &c.

The number below the line, which gives the NAME *to the fraction, is called the* DENOMINATOR.

The number above the line, which expresses the NUMBER *of parts used, is called the* NUMERATOR.

In the fraction $\frac{5}{6}$, the denominator, 6, expresses that some thing or number has been divided into *six equal parts;* and the numerator, 5, expresses that *five of those parts* are taken. The fraction, then, is *five sixths.*

Write the following fractions in figures. Three *fourths;* seven *eighths;* ten *twelfths;* fifteen *sixteenths;* nineteen *twenty-fifths;* twenty-five *thirty-fifths;* thirty-nine *forty-eighths;* fifty-seven *seventy-fifths;* eighty-nine *ninetieths;* one hundred five *one hundred twenty-fifths;* two hundred thirty-five *three hundred forty-fifths;* five hundred fifty-five *seven hundred seventy-sevenths;* eight hundred eighty nine *nine hundred eighty-eighths.*

SECTION II.

Proper and Improper Fractions.

If the value of a fraction be less than a unit, that is, if the numerator be less than the denominator, *it is a* PROPER FRACTION; as $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{10}{12}$, &c.

If the value of a fraction be equal to, or greater than a unit, that is, if the numerator be equal to or greater than the denominator, *it is an* IMPROPER FRACTION; as $\frac{3}{3}$, $\frac{5}{4}$, $\frac{9}{8}$, $\frac{12}{12}$, $\frac{25}{5}$, &c.

Write the following fractions in figures; and separate the proper from the improper fractions.

Two thirds; three halves; four thirds; three fifths; six fourths; eight sevenths; nine elevenths; seven sixths five eighths; five fifths; fifteen sixteenths; seventeen eighths; five twenty-fifths; twenty-five fifths; twenty-five twenty-fifths; ninety-eight eighty-ninths; eighty-nine ninety-eighths.

Although fractions are properly *parts of a unit*, or one single thing, they may be used to express a part or parts of any number of things: thus we may express two thirds of *ten dollars*, as well as two thirds of *one dollar;* or three fourths of *one hundred miles*, as well as three fourths of *one mile.*

SECTION III.

Mixed Numbers.

A whole number with a fraction is called a MIXED NUMBER; as, one and five eighths, $1\frac{5}{8}$; two and three fourths, $2\frac{3}{4}$.

Express the following mixed numbers in figures.

Four and two thirds; six and four fifths; five and one half; seven and four sixths; nine and three sevenths; thirteen and seven eighths; twenty and eleven twelfths; thirty-seven and fifteen sixteenths.

Any number may be written in the form of a fraction: thus, $5 = \frac{5}{1}$; $13 = \frac{13}{1}$. These numbers may be expressed with any denominator required, by *multiplying the whole number by the denominator, and placing the product as a numerator over the given denominator.*

Express 5 as a fraction, having 2 for its denominator.

$5 \times 2 = 10.$ ANS. $\frac{10}{2}$.

Write 10 in the form of a fraction, having 5 for its denominator. ANS. $\frac{50}{5}$.

Change 15 to a fraction, with 15 for a denominator.

ANS. $\frac{225}{15}$.

Change 25 to a fraction, having 25 for its denominator.
Ans. $\frac{625}{25}$.

SECTION IV.

Change of Fractions.

It is often necessary to change the form of a fraction or mixed number. This may be done without changing its value.

To change an improper fraction to a whole or mixed number.

Divide the numerator by the denominator: the quotient will be a whole number.

1. Change $\frac{12}{4}$ to a whole number. Ans. 3.
12 divided by 4 = 3.

If there be a remainder, write it over the denominator, and annex it to the quotient.

2. Change $\frac{16}{5}$ to a mixed number.
16 divided by 5 = 3 and 1 fifth remaining. Ans. $3\frac{1}{5}$.

3. Change $\frac{17}{3}$ to a mixed number. Ans. $5\frac{2}{3}$.

Change the following improper fractions to whole or mixed numbers.

4. Thirteen fifths. Ans. $2\frac{3}{5}$.
5. Twelve sevenths. Ans. $1\frac{5}{7}$.
6. Nineteen eighths. Ans. $2\frac{3}{8}$.
7. $\frac{41}{15}$. Ans. $2\frac{11}{15}$.
8. $\frac{39}{13}$. Ans. 3.
9. $\frac{60}{24}$. Ans. $2\frac{12}{24}$.
10. $\frac{29}{4}$. Ans. $7\frac{1}{4}$.
11. Change $\frac{85}{15}$ to a mixed number. Ans. $5\frac{10}{15}$.
12. Change $\frac{101}{13}$ to a mixed number. Ans. $7\frac{10}{13}$.
13. Change $\frac{578}{69}$ to a mixed number. Ans. $8\frac{26}{69}$.
14. Change $\frac{6675}{1335}$. Ans. 5.
15. Change $\frac{1728}{12}$ Ans. 144.

SECTION V.

Change of Mixed Numbers.

To change a mixed number to an *improper fraction.*

Multiply the whole number by the denominator of the fraction, and to the product add the numerator.

1. Change $5\frac{2}{3}$ to thirds. $5 \times 3 + 2 = 17$. Ans. $\frac{17}{3}$.
2. Change $12\frac{7}{8}$ to an improper fraction. Ans. $\frac{103}{8}$.
3. In $35\frac{3}{4}$ how many fourths? Ans. $\frac{143}{4}$.
4. In $78\frac{1}{3}$ how many thirds? Ans. $\frac{235}{3}$.
5. Change $34\frac{13}{17}$ to seventeenths. Ans. $\frac{591}{17}$.
6. Change $64\frac{4}{9}$ to an improper fraction. Ans. $\frac{580}{9}$.
7. Change $275\frac{5}{6}$ to an improper fraction. Ans. $\frac{1655}{6}$.

Change the following mixed numbers to improper fractions.

8. $54\frac{2}{3}$. Ans. $\frac{164}{3}$.
9. $69\frac{1}{2}$. Ans. $\frac{139}{2}$.
10. $75\frac{3}{8}$. Ans. $\frac{603}{8}$.
11. $123\frac{5}{9}$. Ans. $\frac{1112}{9}$.
12. $157\frac{4}{7}$. Ans. $\frac{1103}{7}$.
13. $450\frac{10}{11}$. Ans. $\frac{4960}{11}$.
14. $97\frac{3}{15}$. Ans. $\frac{1458}{15}$.
15. $45\frac{173}{212}$. Ans. $\frac{9713}{212}$.

SECTION VI.

Change of Denominator.

To change a fraction to its *highest denominator.*

Divide both terms of the fraction by any number that will divide them without a remainder.

1. Change $\frac{8}{12}$ to its highest denominator.
   8 and 12 may be divided by 4. Ans. $\frac{2}{3}$.
2. Change $\frac{12}{15}$ to its highest denominator. Ans. $\frac{4}{5}$.

Note. This operation has been usually called "*reducing fractions to their lowest terms,*" but it is more important to consider fractions in relation to the *value* of the parts which they express, than to regard the abstract numbers of which they are composed.

3. Change $\frac{27}{63}$ to its highest denominator. Ans. $\frac{3}{7}$.

4. Change $\frac{144}{224}$ to its highest denominator. Ans. $\frac{9}{14}$.

5. Change $\frac{275}{550}$ to its highest denominator. Ans. $\frac{1}{2}$.

Express the following fractions with their highest denominators.

6. $\frac{216}{324}$. Ans. $\frac{2}{3}$.
7. $\frac{405}{540}$. Ans. $\frac{3}{4}$.
8. $\frac{455}{525}$. Ans. $\frac{13}{15}$.
9. $\frac{625}{750}$. Ans. $\frac{5}{6}$.
10. $\frac{675}{720}$. Ans. $\frac{15}{16}$.
11. $\frac{925}{1008}$. Ans. $\frac{11}{12}$.

SECTION VII.

Common Denominator.

When the denominators of two or more fractions are alike, they are said to have a *common denominator.*

To change fractions having different denominators, to fractions of equal value, having a common denominator.

Multiply all the denominators together for a common denominator.

Multiply each numerator by the denominators of all the other fractions, and place the result over the common denominator.

1. Change $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{5}$ to fractions of equal value, having a common denominator.

$2 \times 3 \times 5 = 30$, the common denominator.
$1 \times 3 \times 5 = 15$, the numerator of the first fraction.
$2 \times 2 \times 5 = 20$, numerator of the second fraction.
$3 \times 2 \times 3 = 18$, numerator of the third fraction.

Ans. $\frac{1}{2} = \frac{15}{30}$; $\frac{2}{3} = \frac{20}{30}$; $\frac{3}{5} = \frac{18}{30}$.

2. Change $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$ to a common denominator.

Ans. $\frac{3}{4}=\frac{144}{192}$; $\frac{5}{6}=\frac{160}{192}$; $\frac{7}{8}=\frac{168}{192}$.

3. Change $\frac{3}{9}$, $\frac{9}{10}$ and $\frac{10}{11}$ to a common denominator.

Ans. $\frac{330}{990}$; $\frac{891}{990}$; $\frac{900}{990}$.

4. Change $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{8}$ to a common denominator.

Ans. $\frac{480}{720}$; $\frac{576}{720}$; $\frac{600}{720}$; $\frac{630}{720}$.

It sometimes happens that fractions may be changed to a *higher common denominator* than the one obtained by the last method. Any number which is divisible by each of the denominators without a remainder, is a common denominator.

In the second example the number 24 is divisible by each denominator without a remainder; 24 then is a common denominator. The numerators may be found by dividing the common denominator by the denominator of each fraction, and multiplying the quotient by the numerator.

$24 \div 4 \times 3 = 18$, for the first numerator.
$24 \div 6 \times 5 = 20$, " second "
$24 \div 8 \times 7 = 21$, " third "

$\frac{3}{4}=\frac{18}{24}$; $\frac{5}{6}=\frac{20}{24}$; $\frac{7}{8}=\frac{21}{24}$.

Change $\frac{5}{6}$, $\frac{3}{8}$, $\frac{7}{9}$ and $\frac{11}{12}$ to their highest common denominator.

To find the highest common denominator;

Divide two or more of the denominators by any number that will divide them without a remainder. Continue to divide till no two of the denominators or quotients can be so divided. Multiply the divisors and quotients together, and the product will be the greatest common denominator

$$\begin{array}{r|rrrr}
3) & 6 & 8 & 9 & 12 \\
\hline
2) & 2 & 8 & 3 & 4 \\
\hline
2) & 1 & 4 & 3 & 2 \\
\hline
 & 1 & 2 & 3 & 1
\end{array}$$

$3 \times 2 \times 2 \times 2 \times 3 = 72$, the highest common denominator.

$72 \div 6 \times 5 = 60$, the numerator of the first fraction.
$72 \div 8 \times 3 = 27$, the numerator of the second fraction.
$72 \div 9 \times 7 = 56$, the numerator of the third fraction.
$72 \div 12 \times 11 = 66$, the numerator of the fourth fraction.

Ans. $\frac{5}{6} = \frac{60}{72}$; $\frac{3}{8} = \frac{27}{72}$; $\frac{7}{9} = \frac{56}{72}$; $\frac{11}{12} = \frac{66}{72}$.

CHAPTER II.

SECTION I.

Addition of Fractions.

Change the fractions to a common denominator; then add their numerators, and place the result over the common denominator.

1. Add together $\frac{1}{2}$ and $\frac{3}{4}$.

$\frac{1}{2} = \frac{4}{8}$; $\frac{3}{4} = \frac{6}{8}$; $\frac{4}{8}$ and $\frac{6}{8} = \frac{10}{8} = 1\frac{2}{8}$ Ans.

2. What is the sum of $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$?

12 is a common denominator to these fractions, because 12 is divisible by 2 3 and 4.

$\frac{1}{2} = \frac{6}{12}$; $\frac{2}{3} = \frac{8}{12}$; $\frac{3}{4} = \frac{9}{12}$.

$\frac{6}{12} + \frac{8}{12} + \frac{9}{12} = \frac{23}{12} = 1\frac{11}{12}$. Ans. $1\frac{11}{12}$.

3. Add $\frac{3}{4}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{2}$ and $\frac{5}{6}$. Ans. $3\frac{3}{8}$.

4. What is the sum of $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{4}{7}$? Ans. $2\frac{13}{84}$.

5. Add together $2\frac{1}{2}$, $3\frac{3}{4}$ and $4\frac{4}{5}$. Ans. $11\frac{1}{20}$.

Add the whole numbers and fractions separately.

6. How many inches in $2\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{2}{3}$, $5\frac{3}{4}$ and $6\frac{7}{8}$ inches?

Ans. $23\frac{1}{24}$ inches

7. How many feet in $7\frac{1}{2}$, $8\frac{2}{8}$, $10\frac{4}{5}$, $12\frac{3}{8}$ and $13\frac{4}{8}$ feet?
Ans. $52\frac{4}{5}$ feet.

8. How many yards in $5\frac{3}{4}$, $4\frac{3}{8}$, $4\frac{1}{2}$ and $\frac{6}{16}$ yards?
Ans. 15 yards.

SECTION II.

Subtraction of Fractions.

Change the fractions to a common denominator, and subtract one numerator from the other.

1. Subtract $\frac{4}{7}$ from $\frac{3}{5}$.

Change the fractions to thirty-fifths.

$\frac{4}{7} = \frac{20}{35}$ and $\frac{3}{5} = \frac{21}{35}$. $\frac{21}{35}$ minus $\frac{20}{35} = \frac{1}{35}$. Ans. $\frac{1}{35}$.

2. What is the difference between $\frac{2}{3}$ and $\frac{3}{4}$?
Ans. $\frac{1}{12}$.

3. From $10\frac{3}{5}$ take $7\frac{3}{20}$.

Note. Change $\frac{3}{5}$ to twentieths. Ans. $3\frac{9}{20}$.

4. From $29\frac{15}{16}$ dollars take $10\frac{3}{4}$ dollars.
Ans. $19\frac{3}{16}$ dollars.

5. What is the difference between $\frac{1}{2}$ of a yard and $\frac{15}{16}$ of a yard? Ans. $\frac{7}{16}$ of a yard.

6. From a board 26 feet long, a carpenter cut $19\frac{1}{2}$ feet and $4\frac{3}{8}$ feet. How much remained? Ans. $2\frac{1}{8}$ ft.

7. From a piece of cloth measuring $23\frac{7}{8}$ yards, were cut three suits of clothes, $3\frac{1}{2}$ yards each suit. How many yards were left? Ans. $13\frac{3}{8}$ yards.

8. Which is the greater, $\frac{23}{32}$ or $\frac{32}{23}$?
Ans. $\frac{32}{23}$ is greater than $\frac{23}{32}$ by $\frac{495}{736}$.

9. What is the difference between $\frac{67}{100}$ of a dollar and $\frac{2}{3}$ of a dollar? Ans. $\frac{1}{300}$ of a dollar.

10. From $\frac{2}{3}$ of a yard subtract $\frac{2}{3}$ of a foot.

$\frac{2}{3}$ of a yard, or $\frac{2}{3}$ of 3 feet $=$ 2 feet. Ans. $1\frac{1}{3}$ ft.

SECTION III.

Multiplication of Fractions.

To multiply a fraction and whole number together.

Multiply the numerator of the fraction by the whole number: or divide the denominator, when it can be done without a remainder.

1. Multiply $\frac{7}{8}$ by 3. $7 \times 3 = 21$. ANS. $\frac{21}{8} = 2\frac{5}{8}$.

2. Multiply $\frac{5}{16}$ by 4.

In this example the denominator can be divided by 4.

$16 \div 4 = 4$ ANS. $\frac{5}{4} = 1\frac{1}{4}$.

3. What is $\frac{2}{5}$ of 12? ANS. $4\frac{4}{5}$.

4. What is $\frac{6}{13}$ of 15? ANS. $6\frac{12}{13}$.

5. What is $\frac{3}{75}$ of 25? ANS. 1.

6. Multiply 100 by $\frac{4}{25}$. ANS. 16.

7. What is $\frac{4}{25}$ of 100? ANS. 16.

8. Multiply $7\frac{2}{3}$ by 7. ANS. $53\frac{2}{3}$.

9. How many yards of cloth in 8 pieces, each piece measuring $23\frac{3}{4}$ yards? ANS. 190 yards.

10. How many feet long is a row of 9 houses, each measuring $25\frac{2}{3}$ feet in front? ANS. 231 feet.

In the following examples the fraction may be multiplied by dividing the denominator.

11. Multiply $\frac{17}{25}$ by 5. $\frac{17}{25} \times 5 = \frac{17}{5} = 3\frac{2}{5}$. ANS.

12. Multiply $\frac{15}{48}$ by 16. ANS. 5.

13. How much is 8 times $10\frac{5}{16}$? ANS. $82\frac{1}{2}$.

14. Multiply $\frac{25}{275}$ by 25. ANS. $2\frac{3}{11}$.

15. What is the product of $\frac{13}{144}$ multiplied by 12?
ANS. $1\frac{1}{12}$.

SECTION IV.

Multiplication of Fractions.

To multiply two or more fractions together.

Multiply the numerators together for the numerator of the product; and the denominators together for the denominator of the product.

1. Multiply $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ together.

$1 \times 2 \times 3 \times 4 = 24$ the numerator.

$2 \times 3 \times 4 \times 5 = 120$ the denominator.

Ans. $\frac{24}{120} = \frac{1}{5}$.

2. Multiply $\frac{7}{8}$ by $\frac{3}{4}$. Ans. $\frac{21}{32}$.

3. What is $\frac{4}{6}$ of $\frac{15}{30}$? Ans. $\frac{1}{3}$.

4. What is $\frac{1}{3}$ of $\frac{27}{9}$? Ans. 1.

The "multiplication of fractions" is in fact division; and some of the operations may actually be performed by division. In the last example, the numerator 27, may be divided by 3. The next example is the same as the last.

5. Divide $\frac{27}{9}$ by 3. Ans. 1.

6. What is $\frac{5}{6}$ of $\frac{24}{40}$?

See what one sixth is, then $\frac{5}{6}$ will be 5 times as much.

Ans. $\frac{20}{40} = \frac{1}{2}$.

7. Multiply $\frac{2}{3}$, $\frac{3}{8}$, $\frac{4}{5}$ and $\frac{4}{7}$ together. Ans. $\frac{4}{35}$.

8 What is the product of $\frac{3}{4}$ of $\frac{4}{6}$ multiplied by $\frac{4}{5}$ of $\frac{7}{8}$?

Ans. $\frac{7}{20}$.

SECTION V.

Division of Fractions.

To divide a whole number by a fraction: that is, to find how many times a fraction is contained in a whole number.

Multiply the whole number by the denominator and divide the product by the numerator.

1. Divide 12 by $\frac{3}{4}$, that is, see how many times $\frac{3}{4}$ are contained in 12.

$12 \times 4 \div 3 = 16.$ ANS. 16 times.

2. How many times are $\frac{2}{5}$ of a dollar contained in ten dollars? ANS. 25 times.

3. At $\frac{3}{8}$ of a dollar a yard, how many yards of cloth can be bought for 15 dollars? ANS. 40 yards.

4. If a man can travel $7\frac{2}{5}$ miles in an hour, how long will it take him to travel 35 miles? ANS. $4\frac{27}{37}$ hours.

5. How many rods are there in 54 yards?

ANS. $9\frac{9}{11}$ rods.

6. How many rods in $346\frac{1}{2}$ feet? ANS. 21 rods.

SECTION VI.

Division of Fractions.

To divide a fraction by a fraction; that is, to find how many times one fraction is contained in another.

Multiply the numerator of the dividend by the denominator of the divisor, for the numerator of the quotient. Then multiply the denominator of the dividend by the numerator of the divisor, for the denominator of the quotient.

1. Divide $\frac{7}{8}$ by $\frac{2}{3}$; that is, see how many times $\frac{2}{3}$ are contained in $\frac{7}{8}$. $\frac{2}{3}$ the divisor; $\frac{7}{8}$ the dividend.

$7 \times 3 = 21$, numerator. $8 \times 2 = 16$, denominator.

Ans. $\frac{21}{16} = 1\frac{5}{16}$.

2. How many times are $\frac{4}{5}$ contained in $\frac{5}{12}$?

Ans. $\frac{25}{48}$.

3. How many times are $\frac{11}{12}$ contained in $\frac{15}{16}$?

Ans. $1\frac{4}{176}$.

4. Into how many pieces can a board $3\frac{3}{4}$ feet long be cut, each piece measuring $\frac{3}{4}$ of a foot? Ans. 5 pieces.

If the fractions have a common denominator, one numerator may be divided by the other.

In the last example $3\frac{3}{4}$ are equal to $\frac{15}{4}$; and $\frac{3}{4}$ are contained in $\frac{15}{4}$ five times.

5. In $4\frac{7}{8}$ yards, how many times $\frac{3}{8}$ of a yard?

Ans. 13 times.

A fractional part of any number being given, to find the whole number.

This is the same in principle and operation as the last examples.

1. 10 is $\frac{3}{8}$ of what number?

$10 \times 8 = 80 \div 3 = 26\frac{2}{3}$. Ans. $26\frac{2}{3}$.

2. $\frac{3}{7}$ are $\frac{2}{3}$ of what number? Ans. $\frac{9}{14}$.

3. $\frac{7}{8}$ of a dollar are $\frac{3}{4}$ of how many dollars? Ans. $1\frac{1}{6}$.

4. To how many persons can I give 5 dollars, if each one receives $\frac{5}{16}$ of a dollar?

5 dollars are equal to $\frac{80}{16}$; and $\frac{5}{16}$ are contained in $\frac{80}{16}$ 16 times. Ans. 16 persons.

5. Into how many pieces, each $1\frac{7}{8}$ feet long, can a board be divided, $\frac{3}{15}$ of the length of which are equal to five of the pieces? Ans. 25 pieces.

6. What is the length of a board $\frac{3}{5}$ of which measures $15\frac{3}{4}$ feet? ANS. $26\frac{1}{4}$ feet.

SECTION VII.

To find the ratio of two fractions, or of a fraction and whole number; that is to find what part of one fraction or whole number, another fraction is.

Change all the numbers to fractions having a common denominator; and the numerators will express the proportion.

1. What part of $\frac{7}{8}$ is $\frac{2}{3}$?

Change them to twenty-fourths.

$\frac{7}{8}=\frac{21}{24}$; and $\frac{2}{3}=\frac{6}{24}$. ANS. $\frac{16}{21}$.

2. What part of 2 feet are $\frac{3}{8}$ of one foot? ANS. $\frac{3}{16}$.

3. What part of a foot are $\frac{3}{4}$ of an inch? ANS. $\frac{1}{16}$.

4. What part of a yard are $2\frac{1}{2}$ feet? ANS. $\frac{5}{6}$.

5. What part of a rod is one yard and a half? ANS. $\frac{3}{11}$.

6. What part of a rod are three feet? ANS. $\frac{6}{11}$.

7. What part of a mile are $20\frac{1}{2}$ rods? ANS. $\frac{41}{640}$.

8. What is the ratio of $2\frac{1}{2}$ to 7? ANS. $\frac{5}{14}$.

9. What is the proportion of $3\frac{1}{4}$ to 8? ANS. $\frac{13}{32}$.

10. What part of 13 is $\frac{5}{8}$? ANS. $\frac{5}{104}$.

CHAPTER III.

DECIMALS.

SECTION I.

In Decimal Fractions, the denominators are *tenths*, *hundredths*, *thousandths*, &c. The denominators are not usually expressed in figures; the *numerators* only being written, with a point or period at the left, to distinguish them from whole numbers.

The first figure on the right of the point is *tenths*, the second *hundredths*, the third *thousandths*, &c. decreasing in a tenfold ratio. Thus,

Three tenths	$.3 = \frac{3}{10}$.
One tenth, and four hundredths, or fourteen hundredths.	$.14 = \frac{14}{100}$.
Two tenths, three hundredths and four thousandths, or two hundred thirty-four thousandths.	$.234 = \frac{234}{1000}$.
Five thousand seven hundred sixty-eight ten thousandths.	$.5768 = \frac{5768}{10000}$.

In the following table the names of the decimals are expressed as far as billionths.

.3	5	7	4	8	6	9	2	1
Tenths	Hundredths	Thousandths	Tenths of Thousandths	Hund. of Thousandths	Millionths	Tenths of Millionths	Hund. of Millionths	Billionths

It will be perceived that the *denominator* of a decimal fraction is a unit with zeros annexed; and that the number of zeros corresponds to the number of figures in the numerator. Thus in the decimal .25397, the denominator is 1 with five zeros = 100000. The fraction is twenty five thousand three hundred ninty-seven, one hundred thousandths = $\frac{25397}{100000}$.

Write the following decimal fractions in figures.

1. Six tenths.
2. Twenty-five hundredths.
3. One hundred and two thousandths.
4. Nine hundred eighty-seven thousandths.
5. Eight tenths.
6. Sixteen hundredths.
7. Two hundred thirteen thousandths.
8. Seven hundredths.
9. Four thousandths.
10. Thirty-four thousandths.
11. One tenth and two hundredths.
12. Twelve hundredths.
13. Two tenths, three hundredths, and four thousandths.
14. Two hundred thirty-four thousandths.
15. Five tenths.

16. Five hundredths.
17. Five thousandths.
18. Five ten thousandths.
19. Five one hundred thousandths.
20. Five millionths.
21. What is the sum of the first ten of the above examples? ANS. 3.220.
22. What is the sum of the last ten of the above examples? ANS. 1.263555.

Decimal Fractions decrease from left to right in the same ratio as whole numbers, and are governed by the same principles. Therefore Addition, Subtraction, Multiplication and Division of Decimals are the same as in whole numbers. Attention must be paid to placing the decimal point.

Write the following whole numbers and decimals in figures.

1. Two, and five tenths.
2. Three, and six hundredths.
3. Five, and twenty-five thousandths.
4. Ten, and nine tenths.
5, Nine, and nineteen hundredths.
6. The amount of the last five examples is thirty, and six hundred seventy-five thousandths. 30.675.

SECTION II.

Change of Common Fractions to Decimals.

Annex zeros to the numerator, and divide the product by the denominator.

1. Change $\frac{1}{4}$ to a decimal.

4) 100

.25 ANS. .25.

2. Change the following common fractions to decimals. $\frac{1}{2} = .5$; $\frac{3}{4} = .75$; $\frac{3}{5} = .6$; $\frac{7}{8} = 875$.

Express the fractions in the following numbers in decimals.

3. Twelve, and one half. ANS. 12.5
4. Six, and one quarter. ANS. 6.25
5. Eighteen, and three fourths. ANS. 18.75
6. Five, and three eighths. ANS. 5. 375.

SECTION III.

Addition of Decimals.

Place the decimals, tenths under tenths, hundredths under hundredths, &c. and perform the addition as in whole numbers: from the result separate as many figures for decimals as are equal to the greatest number in any of the given fractions.

1. Add together seven tenths, fourteen hundredths, and thirteen thousandths.

.7
.14
.013

.853 thousandths.

2. Add together two and five tenths, three and twenty-five hundredths, and four hundred fifty-six thousandths.

ANS. 6.206.

3. Add together 24.16, 85.213, 27.08, and 9.231.
Ans. 145.684.

4. Add together, $2\frac{1}{4}$ feet, $4\frac{1}{6}$ feet, $5\frac{1}{3}$ feet, $6\frac{3}{4}$ feet, and $7\frac{3}{8}$ feet. Change the fractions to decimals.
Ans. 25.875.

5. How many are $10\frac{1}{2}$, $5\frac{2}{5}$, $6\frac{7}{8}$, $4\frac{3}{4}$ and $2\frac{1}{8}$?
Ans. 29.65.

6. How many feet are there in four boards measuring $35\frac{1}{4}$ feet, $27\frac{1}{8}$, $40\frac{4}{5}$, and $60\frac{3}{4}$ feet?
Ans. 163.925 feet.

SECTION IV.

Subtraction of Decimals.

Subtract as in whole numbers; and separate the decimal places as in addition.

1. What is the difference between five tenths, and five hundredths?

$$\begin{array}{r} .5 \\ .05 \\ \hline .45 \end{array}$$

Ans. 45.

2. Subtract seventy-five hundredths from nine tenths.
Ans. .15.

3. How much more is one tenth than one thousandth?
Ans. .099.

4. From fifty hundredths, take fifty thousandths.
Ans. .450.

5. From one hundred, and fifty-four hundredths, subtract one hundred, and fifty-four thousandths.
Ans. .486.

6. Subtract five hundred twenty-five thousandths from two hundred fifteen. Ans. 214.475.

7. Take three, and seven tenths, from six, and nine hundredths. Ans. 2.39.

8. From a board 25. 873 feet in length, a carpenter cut off a piece which measured 9.876 feet; how much remained? Ans. 15.997 feet.

9. From a farm containing 25 acres, a farmer sold three tenths of an acre more than one half of the whole, how much had he left? Ans. 12.2.

SECTION V.

Multiplication of Decimals.

Multiply as in whole numbers; and from the right of the product separate as many decimals as are equal to the decimals in both factors.

1. Multiply five tenths by four tenths.

$.5 \times .4 = .20.$ Ans. .20.

2. Multiply eight and seven tenths by nine and eight tenths. Ans. 85.26.

3. Multiply two and fifteen hundredths by one and three tenths. Ans. 2.795.

If there are not as many figures in the product as are equal to the decimals in both factors, supply the deficiency by prefixing zeros.

4. Multiply four tenths by fourteen hundredths.

$4 \times .14 = .56.$ as there are *three* decimals in the two factors, the product must contain *three* also. A zero must be prefixed to the product. Ans. .056.

5. At three cents per foot, how much will 387.51 feet of boards come to?

One cent is one hundredth of a dollar; three cents are three hundredths of a dollar = .03. Ans. $11.6253.

6. What is the cost of painting a room containing 258.72 square yards, at 12½ cents per yard? Ans. $32.34.

7. What will be the expense of glazing 54 windows, each having 12 lights, at .125 of a dollar per light? Ans. 81 dollars.

SECTION VI.

Division of Decimals.

Divide as in whole numbers, and from the right of the quotient separate as many decimals, as the number of decimals in the dividend exceed those in the divisor. If the quotient does not contain the requisite number of figures, supply the deficiency by prefixing zeros.

1. Divide 18.25 by 5. Ans. 3.65.
2. Divide 3.75 by 3. Ans. 1.25.
3. Divide 264.75 by 2. 5. Ans. 10 .9.
4. If 2.25 yards of cloth will make a coat, how many coats may be made from twenty-seven yards? Ans. 12 coats.

If the number of decimals in the divisor and dividend are equal, the quotient is a whole number.

5. Divide 88.2 by 9.8. Ans. 9.
6. How many thousands of boards at 7⅛ dollars per thousand, can be bought for $385.75 cents? Ans. 54.

7. At 8½ cents per foot, how many feet of mahogany can be bought for eighteen dollars 27½ cents? Ans. 215 feet.

8. How many square yards of painting at 8¼ cents per yard can be done for $15.26¼? Ans. 185 yds.

9. How many feet of timber at $0.205 per foot, may be bought for 37 dollars 72 cents? Ans. 184 feet.

CHAPTER IV.

DUODECIMALS.

Arithmetical operations upon Feet, Inches, and Twelfths are called Duodecimals; because the denominators increase in a twelvefold ratio.

SECTION I.

Addition of Duodecimals.

Add as in whole numbers, and divide the sum of each denomination, except feet, by 12.

1. Add together 3 feet 6½ inches; 4 feet 3¼ inches, and 6 feet 4¾ inches.

Change the fractions of inches to twelfths. $\frac{1}{2}=6$ twelfths, $\frac{1}{4}=3$ twelfths, $\frac{3}{4}=9$ twelfths.

Feet.	inches.	12ths.
3	6	6
4	3	3
6	4	9
14	2	6

Ans. 14 feet $2\frac{1}{2}$ inches.

The fractions of inches may be added by changing them to fourths.

2. What number of feet are there in four boards, the first measuring 24 feet $4\frac{1}{3}$ inches; the second 21 feet $8\frac{1}{2}$ inches; the third 19 feet $6\frac{1}{2}$ inches; and the fourth 25 feet $7\frac{3}{4}$ inches? Ans. 91 feet $3\frac{1}{12}$ inches.

3. How many feet are there in $15\frac{3}{4}$ feet, 24 feet 6 inches, $13\frac{2}{3}$ feet, and 56 feet 9 inches? Ans. $110\frac{2}{3}$ feet.

4. How many feet are $25\frac{3}{6}$ feet, 72 feet $6\frac{3}{4}$ inches, 90 feet $6\frac{1}{2}$ inches, 100 feet $\frac{1}{4}$ inches, and $35\frac{3}{8}$ feet? Ans. 324 feet.

SECTION II.

Subtraction of Duodecimals.

Subtract as in whole numbers. If a number be less than the one below it, increase it by 12; then diminish the next upper number by 1.

1. From 13 feet $10\frac{2}{3}$ inches, take 4 feet $8\frac{1}{2}$ inches.

Feet.	in.	12ths.
13	10	8
4	8	6
9	2	2

Ans. 9 feet $2\frac{1}{6}$ inches.

2. Subtract 7 feet $9\frac{3}{4}$ inches, from 10 feet $8\frac{1}{2}$ inches. Ans. 2 feet $10\frac{3}{4}$ inches.

3. From a timber 42 feet $6\frac{3}{4}$ inches in length, a carpenter sawed off 11 feet $9\frac{1}{3}$ inches, how much remained? Ans. 30 feet $9\frac{5}{12}$ in.

4. What is the difference between 37 yards $2\frac{3}{4}$ feet, and 37 feet $2\frac{3}{4}$ inches? Ans. 76 feet $6\frac{1}{4}$ in.

5. From a board 28 feet $9\frac{1}{2}$ inches in length, a piece was cut off measuring $13\frac{3}{4}$ feet, how long was the part left? Ans. 15 feet $\frac{1}{2}$ inch.

6. From a mast 205 feet long, a part was broken off; the remaining part measured $35\frac{3}{4}$ yards, what was the length of the broken piece? and what part of the whole mast was it? Ans. $97\frac{3}{4}$ feet. $\frac{391}{820}$ of the whole.

SECTION III.

Multiplication of Duodecimals.

Multiply as in whole numbers; and divide the product of each denomination by 12; except in multiplying feet by feet; then add the several products.

1. Multiply 4 feet $5\frac{1}{2}$ inches by 2 feet $7\frac{1}{4}$ inches.

Change the fractions of inches to twelfths. $\frac{1}{4}$ is 3 twelfths; $\frac{1}{2}$ is 6 twelfths.

		feet.	in.	12ths.
		4	5	6
		2	7	3
	1	1	4	6
2	7	2	6	
8	11	0		
11	7	3	10	6

11 Square feet and $\frac{7}{12}$ $\frac{3}{144}$ $\frac{10}{1728}$ $\frac{6}{20736}$ of a square foot. Ans. 11 square feet, $87\frac{7}{8}$ square inches.

In this example the first number on the left of the result is 11 square feet, the second number 7, is not inches; but twelfths of a square foot. One twelfth of a square foot is 12 square inches, and seven twelfths are 84 square inches. The third number 3, is three one hundred forty-fourths of a foot, three square inches, which added to 84 square inches are 87 square inches, the fourth number is twelfths of a square inch, and the fifth is one hundred forty-fourths of a square inch.

2. Multiply 2 feet $6\frac{3}{4}$ inches by 5 feet $8\frac{1}{3}$ inches.
Ans. 14. $85\frac{1}{4}$ square feet.

3. Multiply 8 feet $2\frac{2}{3}$ inches by 4 feet $8\frac{1}{4}$ inches.
Ans. $38\frac{11}{24}$ sq. feet.

4. Multiply 21 feet $6\frac{1}{2}$ inches by 7 feet $9\frac{1}{4}$ inches.
Ans. 167 sq. feet $57\frac{1}{8}$ sq. inch.

5. Multiply 2 feet $3\frac{1}{4}$ inches by 5 feet $6\frac{1}{2}$ inches.
Ans. 12 sq. feet, $84\frac{1}{8}$ sq. inches.

6. What is the product of 6 feet $8\frac{2}{3}$ inches multiplied by 7 feet $9\frac{3}{4}$ inches. Ans. 52 sq. feet, $74\frac{1}{2}$ sq. inches.

7. What is the length of a fence enclosing a square each side of which is 17 feet $9\frac{1}{3}$ inches?

Ans. 71 feet, $1\frac{1}{3}$ inches.

8. How many square feet are there in 9 boards, each measuring 23 feet $7\frac{3}{4}$ inches?

Ans. 212 sq. feet $9\frac{3}{4}$ in.

In the last two examples the multiplier is an integer, or whole number, and the denominators of the multiplicand are not reduced.

SECTION IV.

Division of Duodecimals.

Divide each denomination as in whole numbers. Multiply the remainder, if any, by 12, and to the product add the next denomination, and divide as before.

1. Divide 14 feet $3\frac{1}{4}$ inches by 5.

	Ft.	in.	12ths.
5)	14	3	3
	2	10	3

Ans. 2 feet $10\frac{1}{4}$ inches.

2. Divide 17 feet $4\frac{2}{3}$ inches into 8 equal parts.

Ans. 2 feet $2\frac{1}{12}$ inches.

3. From a board 23 feet $7\frac{1}{4}$ inches in length, a carpenter cut one third of it, how much did the piece remaining measure? **Ans.** 15 feet $8\frac{5}{6}$ inches.

4. A man wishes to cut a board, measuring 21 feet $8\frac{1}{2}$ inches in length, into four equal parts; how much will each piece measure? **Ans.** 5 feet, $5\frac{1}{8}$ inches.

5. Divide a piece of land containing 225 square feet 10 square inches, into seven equal parts, how many feet will each piece contain? Ans. 32 sq. feet, 22 sq. in.

Note. — After dividing the feet by 7 the one sq. foot remaining is = 144 square inches.

6. What part of 1 foot is 9 inches? Ans. $\frac{3}{4}$.

7. What part of 1 foot 3 inches is 6 inches. Ans. $\frac{2}{5}$.

8. What part of 6 feet 8 inches is 3 feet 9 inches? Ans. $\frac{9}{16}$.

9. Divide 16 feet $3\frac{3}{4}$ inches by 5 feet $5\frac{1}{4}$ inches. That is, find how many times the one is contained in the other. Change them to fourths. Ans. 3.

10. Into how many pieces of 3 feet $6\frac{3}{4}$ inches each, can a board be cut, which measures 17 feet $9\frac{3}{4}$ inches? Ans. 5 pieces.

CHAPTER V.

INVOLUTION.

SECTION I.

Involution is the continual multiplication of the product of some number by itself.

The number itself is called the *first power or root:* and the successive products are called the *second, third, fourth powers,* &c.

1. Thus let 5 be the first power or root.

$5 \times 5 = 25$ the square or second power.

$5 \times 5 \times 5 = 125$ the cube or third power.

$5 \times 5 \times 5 \times 5 = 625$ the biquadrate or fourth power.

2. What is the square or second power of 25? **Ans.** 625.

3. What is the square of 75? **Ans.** 5625.

4. What is the square of 245? **Ans.** 60025.

5. What is the cube of 95? **Ans.** 857375.

6. What is the cube of 123? Ans. 1860867.

7. What is the biquadrate of 25? **Ans.** 390625.

8. What is the square, cube, and biquadrate of 125?
Ans. square 15625.
cube 1953125.
biquadrate 244140625.

Table of Powers.

Roots or First Powers.	1	2	3	4	5	6	7	8	9	10
Squares or 2nd. Powers.	1	4	9	16	25	36	49	64	81	100
Cubes or 3d Powers.	1	8	27	64	125	216	343	512	729	1000
Biquadrates or 4th. Powers.	1	16	81	256	625	1296	2401	4096	6561	10000

SECTION II.

Evolution: or Extraction of the Roots.

Evolution is finding the root or first power of any given number.

Thus, 5 is the square root or first power of 25. 9 is the cube root or first power of 729.

The operation of finding the first power is called "Extracting the root."

This principle is of great practical utility in its application to the laws of philosophy and mechanics.

SECTION III.

Extraction of the Square Root.

1. What is the square root of 1225?

Point off successively every two figures in the given number beginning at the right?

12.25

Find the greatest square in the left hand period, subtract it therefrom, and place its root in the quotient.

12.25 (3
9
3

Bring down the next period to the right of the remainder, for a new dividend, and double the quotient for a new divisor.

12.25 (3
9
6) 325.

Find how many times this divisor is contained in the qrst two figures of the dividend, place the result in the fiuotient and also at the right of the divisor.

12.25 (35
9
65) 325

Multiply the divisor by the second figure of the quotient and subtract the product from the dividend.

12.25 (35
9
65) 325
325

When there are more figures in the dividend, bring down each successive period, and proceed as before.

2. What is the square root of 256?

2.56 (16
1
26) 156
156
Ans. 16.

3. What is the square root of 676?

6.76 (26
4
46) 276
276
Ans. 26.

4. What is the square root of 9801?

98.01 (99
81
189) 1701
1701
Ans. 99.

5. Extract the square root of 5625. Ans. 75.
6. Extract the square root of 105625. Ans. 325.
7. What is the square root of 213444? Ans. 462.
8. What is the square root of 425104? Ans. 652.
9. What is the square root of 633616? Ans. 796.
10. Find the square root of 974169. Ans. 987.
11. Find the square root of 10725625. Ans. 3275.
12. What is the square root of 21455424. Ans. 4632.

SECTION IV.

Square Root of Fractions.

Change the fractions to their highest denominators, then *extract the square root of the numerator, and the square root of the denominator, and they will express the square root of the fraction.*

1. What is the square root of $\frac{169}{676}$? Ans. $\frac{13}{26}=\frac{1}{2}$.
2. What is the square root of $\frac{625}{5625}$. Ans. $\frac{25}{75}=\frac{1}{3}$.
3. What is the square root of $\frac{225}{576}$. Ans. $\frac{15}{24}=\frac{5}{8}$.

SECTION V.

Extraction of the Cube Root.

To extract the cube root of any given number.

Separate the number by pointing off successively three figures beginning at the right.

1. What is the cube root of 42875?

42.875

From the left hand period subtract the greatest cube it contains, and place its root in the quotient.

42.875 (3 The greatest cube in 42 is
27 27, the root of which is 3.
15

Bring down the first figure of the next period and place it at the right of the remainder for a new dividend. Multiply the square of the quotient by 3, for a new divisor; and in the quotient write the number of times it is contained in the dividend.

42.875 (35 The square of 3 is 9; and
27 3 times 9 is 27.
27) 158
135 27 is contained in 158, 5 times.

Subtract the cube of the quotient from the first two periods of the dividend.

The cube of 35 is 42875, there will then be no remainder; and as there are no more figures in the dividend the work is completed, and 35 is the cube root required.

2. What is the cube root of 12812904?

12.812.904 (234
8
12) 48
12167
1587) ..6449
12812904

12 is contained in 48 4 times; but the cube of 24 is greater than the first two periods of the dividend; it must therefore be diminished, and 23 must be used.

The second divisor is obtained by multiplying the square of 23 by 3.

$23 \times 23 \times 3 = 1587.$

Ans. 234.

3. What is the cube root of 44361864?

44.361.864 (354
27
27) 173
42875
3675) 14868
44361864

Ans. 354.

4. What is the Cube Root of 673373097125?

Ans. 8765

SECTION VI.

Cube Root of Fractions.

Change the fraction to its highest denominator, then extract the root of its numerator, and the root of its denominator, and they will express the cube root of the fraction.

1. What is the cube root of $\frac{27}{125}$?

The cube root of 27 is 3, and the cube root of 125 is 5.
ANS. $\frac{3}{5}$.

2. What is the cube root of $\frac{343}{512}$ ANS. $\frac{7}{8}$.

3. Extract the cube root of $\frac{54}{432}$.

This fraction may be changed to a higher denominator. 54 divided by 2 is 27 ; the cube root of 27 is 3.
432 divided by 2 is 216; the cube root of 216 is 6.
ANS. $\frac{3}{6} = \frac{1}{2}$.

SECTION VII.

Proportionals.

A. To find a mean proportional between two numbers.

Multiply the two numbers together, and the square root of the product is the proportional required.

1. What is the mean proportional between 8 and 18? $18 \times 8 = 144$, the square root of which is 12. ANS. 12.

2. What is the mean proportional between 12 and 27? ANS. 18.

3. What is the mean proportional between 27 and 75? ANS. 45.

4. Find the mean proportional between $7\frac{1}{2}$ and 30. ANS. 15.

5. Find the mean proportional between 64 and 100. ANS. 80.

6. What is the mean proportional between 72 and 128? ANS. 96.

7. What is the mean proportional between 135 and 375? ANS. 225.

B. To find a proportional to the *less* of two numbers, *Divide the square of the less, by the greater number, the quotient will be the proportional required.*

1. What is the less proportional to 12 and 18?
$12 \times 12 = 144 \div 18 = 8$ the less proportional.

2. What is the less proportional to 36 and 54?
Ans. 24.

3. What number bears the same proportion to 40, that 40 bears to 64? Ans. 25.

C. To find a proportional to the *greater* of two numbers.

Divide the square of the greater by the less number, the quotient will be the proportional required.

1. What is the proportional greater to 9 and 15?
$15 \times 15 = 225 \div 9 = 25$ the greater proportional.

2. What is the proportional to the greater of 12 and 18? Ans. 27.

3. What number bears the same proportion to 40 which 40 bears to 25? Ans. 64.

D. To find a fourth proportional to the *least* in a series of three numbers.

Divide the product of the first two by the last number, the quotient will be the proportional.

1. What is the fourth less proportional to 5, 8, and 12? $5 \times 8 = 40 \div 12 = 3\frac{1}{3}$. Ans. $3\frac{1}{3}$.

2. What is the fourth less proportional to 9, 18 and 27? Ans. 6.

3. What number is in the same proportion to 12 which 27 is to 36? Ans. 9.

E. To find a fourth proportional to the *greatest* in a series of three numbers.

Divide the product of the last two by the first number, the quotient will be the proportional required.

1. What is the fourth greater proportional to 13, 15 and 20? $20 \times 15 = 300 \div 13 = 23\frac{1}{13}$. Ans. $23\frac{1}{13}$.

2. What is the fourth greater proportional to 15, 18 and 25? Ans. 30.

3. What number bears the same proportion to 36 which 12 bears to 16? Ans. 27.

CHAPTER VI.

SQUARES OF NUMBERS.

SECTION I.

To find the *sum* of the squares of two numbers without finding the squares of either.

To twice the product of the two numbers, add the square of their difference.

1. What is the sum of the squares of 7 and 17?

$17 \times 7 \times 2 = 238 \qquad 10 \times 10 = 100$

$238 + 100 = 338$ Ans. 338.

2. What is the sum of the squares of 13 and 25?
Ans. 794.

3. What is the sum of the squares of 88 and 98?
Ans. 17348

4. What is sum of the squares of 125 and 175?
Ans. 46250.

Note. This principle is applied in finding the square of the hypothenuse of a right triangle; the base and perpendicular being given.

5. Required the sum of the squares of 60 and 144.
Ans. 24336.

SECTION II.

To find the *difference* of the squares of two numbers, without finding the squares of either.

The sum of two numbers, multiplied by their difference is the difference of their squares.

1. What is the difference between the squares of 15 and 25? $15+25=40$; and $25-15=10$.
$40 \times 10=400$. Ans. 400.

2. What is the difference between the squares of 35 and 95? Ans. 7800.

3. What is the difference between the squares of 224 and 776? Ans. 552000.

4. What is the difference between the squares of $\frac{3}{4}$ and $\frac{7}{8}$? Ans. $\frac{13}{64}$.

This rule is very convenient in finding the square of the base or perpendicular of a right triangle.

It is also applied in finding the difference of the areas of circles and squares.

5. What is the difference between the squares of 72 and 78? $72+78=150 \times 6=900$ Ans. 900.

SECTION III.

To find the the product of the squares of two numbers, without finding the squares of either.

Multiply the two numbers together, and that product by itself, the last is the product of the squares of the two numbers.

1. What is the product of the squares of 4 and 12?

$12 \times 4 = 48 \times 48 = 2304$ Answer.

2. What is the product of the square of 15 multiplied by the square of 9? Ans. 18225.

3. What is the product of the squares of 19 and 57? Ans. 1172889.

4. What is the product of the square of 12 multiplied by the square of 5?

$12 \times 5 = 60 \times 60 = 3600$ Ans. 3600.

5. What is the product of the square of $\frac{2}{3}$ multiplied by the square of $\frac{3}{4}$? Ans. $\frac{1}{4}$.

Note. *It will be perceived by the above examples, that the product of any two numbers is the square root of the product of their squares.*

GEOMETRY.

PART SECOND.

CHAPTER I.

GEOMETRICAL FIGURES AND DEFINITIONS.

SECTION I.

Lines and Angles.

The science of Geometry demonstrates the principles of *extension* and *magnitude;* and is applied to the measuring of lines, surfaces, and solids.

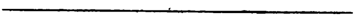
The simplest idea in Geometry is a *point;* which is defined to be, *position without magnitude.*

A point is represented by a small dot. (.)

The motion of a point produces *a line.* A line has only one dimension, namely, length, without breadth or thickness.

The extremities and intersections of lines are points.

A Straight Line.



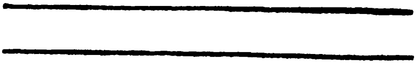
A Straight Line lies wholly in one direction, and is the shortest distance between two points.

A Curved Line.

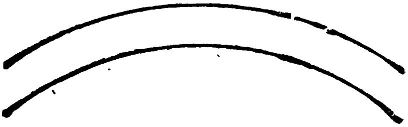
A Curved Line is constantly and uniformly changed in its direction; and is not the shortest distance between two points.

Parallel Lines.

Parallel Straight Lines.



Parallel Curved Lines.

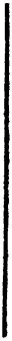


Two or more lines which, extending in the same direction, can never meet, are parallel lines.

A Horizontal Line.

A Line drawn parallel to the horizon is a called a horizontal line.

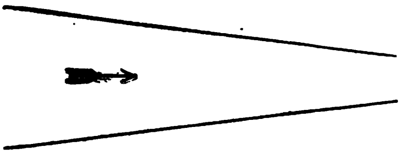
A Vertical Line.



A Line drawn perpendicular to the horizon is a Vertical Line.

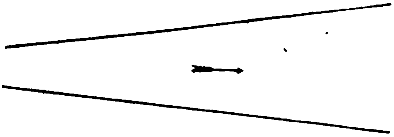
A correct idea of this line is important. It is frequently described only as a perpendicular line. A line may be perpendicular in any direction. But a vertical line can be drawn only in one direction. A string suspending a weight at rest, represents a vertical line. *All vertical lines are perpendicular ; but all perpendicular lines are not vertical.*

Converging Lines.



Two or more lines, if extended in the same direction, approach each other, are *Converging Lines.*

Diverging Lines.

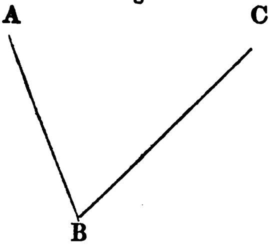


Two or more lines extending in the same direction, which recede from each other, are *diverging lines.*

Define and represent a Geometrical Point; a Straight Line; a Curved Line; Parallel Straight Lines; Parallel Curved Lines; a Horizontal Line; a Vertical Line; Converging Lines; Diverging Lines.

Angles.

An Angle.

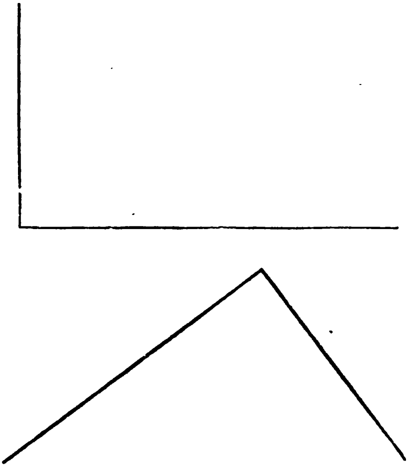


When two straight lines meet in a point, the space or opening between them is called an Angle.

The space between A, B, C, is an angle.

The point B is the *vertex.* The lines A B and B C are the sides of the angle. An angle is not determined by the length of the sides, but by the *space* between them.

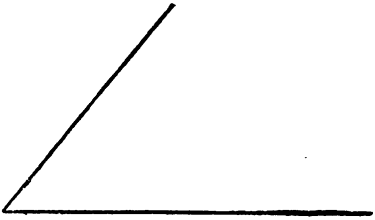
A Right Angle.



If the sides of an angle are perpendicular each to the other, they form a Right Angle, and the space or opening between them contains ninety degrees.

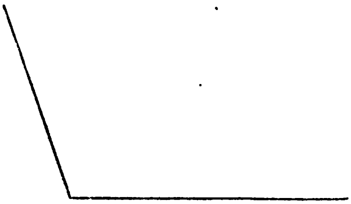
The *position* of an angle does not affect its *quality*. If the sides be perpendicular, the angle is a right angle in whatever position it may be placed.

An Acute Angle.



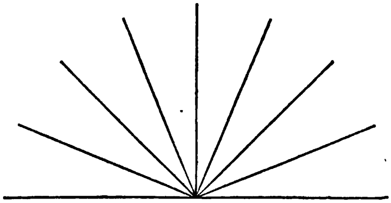
An angle *less than a right angle,* is called an *Acute Angle,* and contains less than ninety degrees.

An Obtuse Angle.

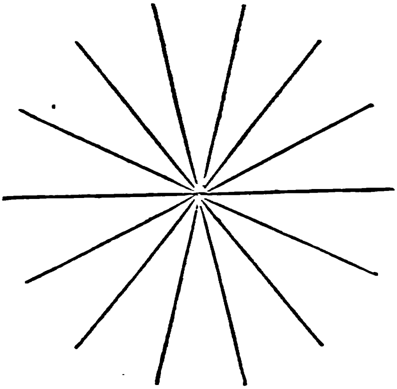


An angle *greater than a right angle,* is called an *Obtuse Angle,* and contains more than ninety degrees.

In the last two angles, the lines forming the sides are oblique each to the other. These angles are sometimes called *Oblique Angles.*

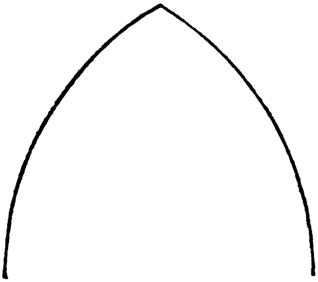


The sum of all the angles which can be formed on the *same side* of a straight line, is equal to two right angles, or 180 degrees, the measure of half a circle.



The sum of all the angles which can be formed *about a given point, or on both sides of a straight line,* is equal to four right angles, or 360 degrees, the measure of a circle.

A Spherical Angle.



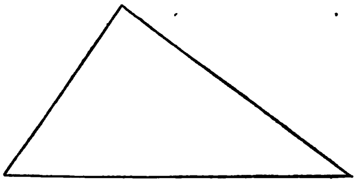
A Spherical Angle is formed by lines drawn on a curved surface.

Define and represent *an Angle; a Right Angle; an Acute Angle; an Obtuse Angle;* how many right angles can be drawn on the same side of a straight line? How many on both sides?

SECTION II.

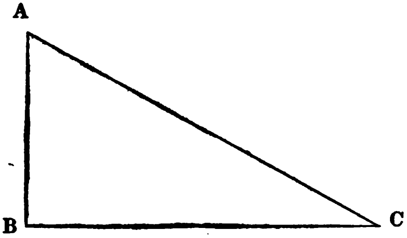
Triangles.

A Triangle.



A figure bounded by *three straight lines,* and having *three angles,* is a *Triangle.*

A Right Triangle.

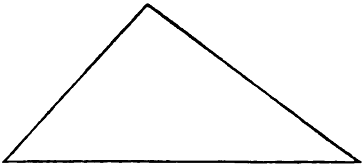


A Triangle that has one right angle is called a *Right Triangle.*

A Triangle cannot have more than *one right angle.*

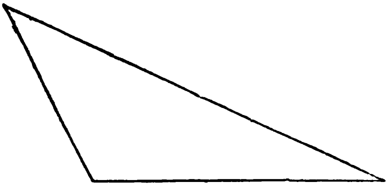
The line A C, opposite the right angle, is the hypothenuse. The line B C, is the base, and the line A B is the perpendicular.

An Acute Triangle.



If all the angles of a triangle *are acute,* it is called an *Acute Triangle.*

An Obtuse Triangle.

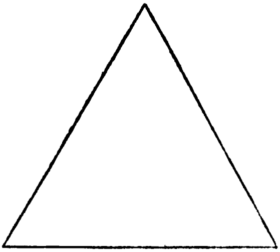


A triangle having *one obtuse angle* is called an *Obtuse Triangle.*

A triangle cannot have more than one *obtuse angle.*

Triangles are also named from the comparative length of their sides.

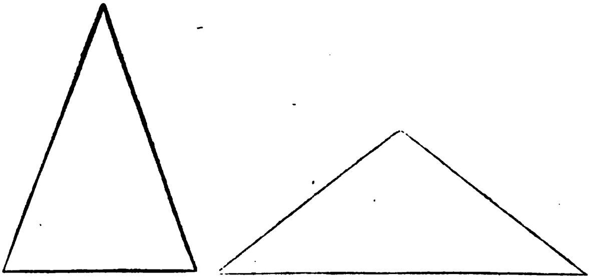
An Equilateral Triangle.



If the three sides of a triangle are equal, it is called an *Equilateral Triangle.*

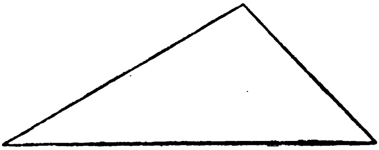
An equilateral triangle is also equiangular.

An Isosceles Triangle.



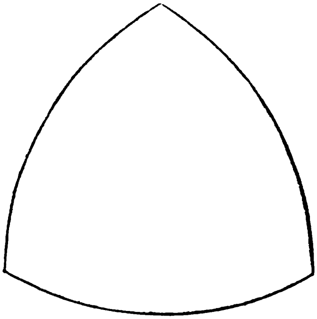
When *two* sides of a triangle are equal, and longer or shorter than the third side, it is called an *Isosceles Triangle.*

A Scalene Triangle.



If the three sides of a triangle are of *unequal length*, it is called a *Scalene Triangle.*

A Spherical Triangle.



Spherical Triangles are formed by lines drawn on curved surfaces.

Describe and draw a Triangle; a Right Triangle; an Obtuse Triangle; an Acute Triangle; a Scalene Triangle; an Equilateral Triangle; an Isosceles Triangle; a Spherical Triangle. Which of the three sides of a right triangle is the hypothenuse? perpendicular? base?

SECTION III.

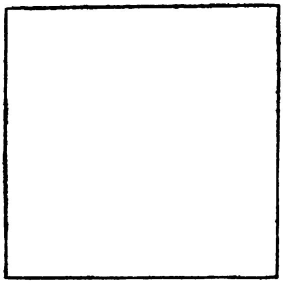
Quadrilaterals.

Figures bounded by four straight lines are called Quadrilaterals.

Parallelograms.

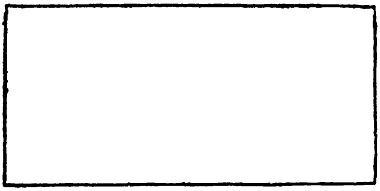
Parallelograms are quadrilateral figures, the opposite sides of which are equal and parallel.

A Square.



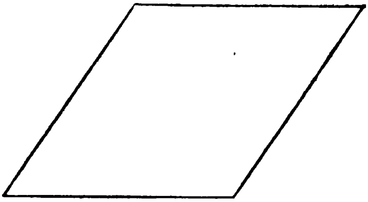
A Square is a parallelogram, having *all its sides equal* and all its angles *right angles.*

An Oblong.



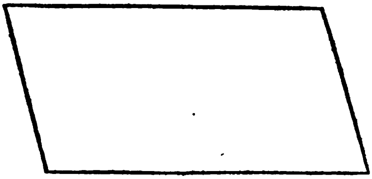
An Oblong is a parallelogram having its *opposite sides equal* and parallel, and its angles *right angles.*

A Rhombus.



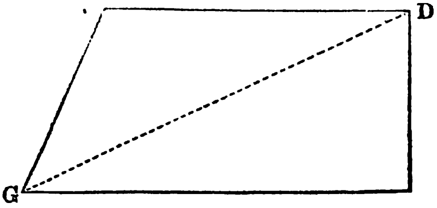
A Rhombus is a parallelogram, having *all its sides equal,* its opposite sides parallel, but its angles not *right angles.*

A Rhomboid.



A Rhomboid is a parallelogram having its opposite sides equal and parallel, and its angles not *right angles.*

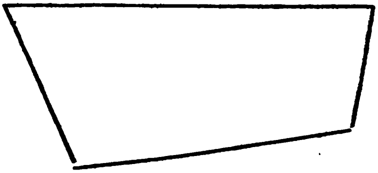
A Trapezoid.



A Trapezoid is a quadrilateral, having only *two* of its sides parallel.

A line drawn through the opposite angles of any plain figure, is a *diagonal.* The line D G is a diagonal.

A Trapezium.



A Trapezium is a quadrilateral, having *none* of its sides parallel.

What are Quadrilateral figures? What are Parallelograms? Draw and explain the properties of a Square; an Oblong; a Rhombus; a Rhomboid; a Trapezoid; a Trapezium; a Diagonal line. Which of these figures are *parallelograms,* and which are not?

SECTION IV.

Multilateral Figures, or Polygons.

Figures having *many sides,* are called *Multilaterals.* They are also called *Polygons,* because they have *many angles.*

Polygons which have equal sides and equal angles, are called regular polygons. Those which have not are called irregular polygons.

A Pentagon has *five sides* and *five angles.*

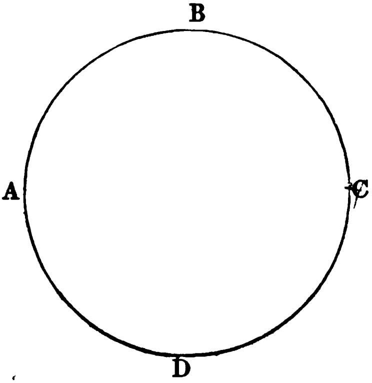
A Hexagon has *six sides* and *six angles.*

A Heptagon has *seven sides* and *seven angles.*
An Octagon has *eight sides* and *eight angles.*
A Nonagon has *nine sides* and *nine angles.*
A Decagon has *ten sides* and *ten angles.*
An Undecagon has *eleven sides* and *eleven angles.*
A Duodecagon has *twelve sides* and *twelve angles.*

The number of sides and angles in a Polygon might be multiplied to infinity. It would then become a *Circle.*

What are Multilateral figures? Why are they called Polygons? What are are Regular Polygons? Irregular Polygons? What is a polygon with eight sides and eight angles called? What is one with ten sides and ten angles called? What is a figure with four equal sides and four equal angles called? What is a figure with three sides and three angles called? What is a polygon with an *infinite* number of sides and angles called?

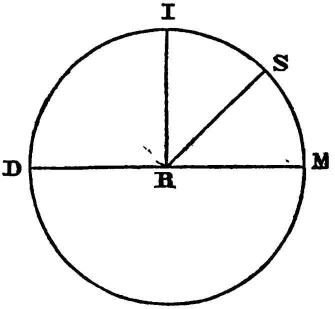
A Circle.



A Circle is a regular polygon with an infinite number of sides and angles, all equally distant from the centre.

The curve line A B C D bounding the figure is called the *periphery*, or circumference. *The space enclosed is the Circle.*

Diameter and Radius.



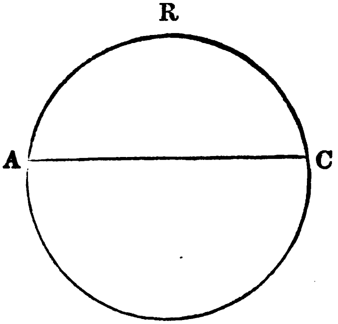
A straight line drawn through the centre of a circle and dividing it into two equal parts, is the *Diameter*.

The line D M is a *Diameter*.

A line drawn from the centre to the circumference a circle is a *Radius*; two or more are *Radii*.

The line R S is a radius. The lines R M, R S, R I, and R D are radii.

Chord and Arc.



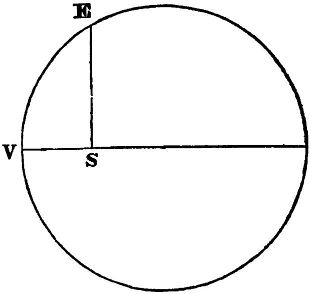
A line drawn through any part of a circle less than a diameter, is called a *Chord.*

The line A C is a *Chord.*

Any part of the circumference is *an Arc of a Circle.*

The curve line A R C is an *Arc.*

Sine and Versed Sine.



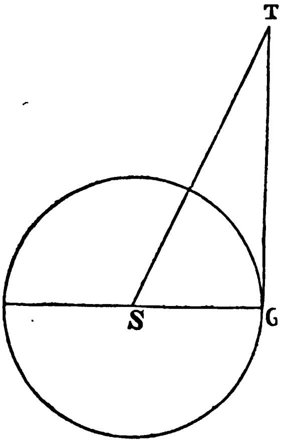
A line drawn from the extremity of an arc perpendicular to the radius or diameter is the *Sine* of an arc.

The part of the radius or diameter uniting with the other extremity of the arc is the *Versed Sine.*

The line S E is the *Sine* of the arc V E.

The line V S is the *Versed Sine* of the arc V E.

Tangent and Secant.



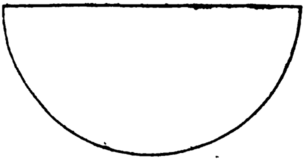
A straight line touching the circumference only in one point without the circle, perpendicular to the diameter is a *Tangent.*

A straight line drawn through the circle, cutting the circumference in one or more points, is a *Secant.*

The line T G is a tangent.

The line S T is a *Secant.*.

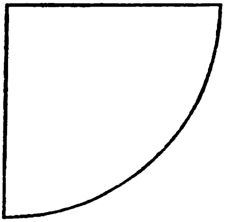
A Semicircle.



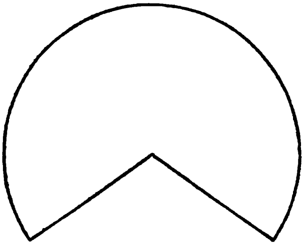
Half a circle is a *Semicircle.* A semicircle is bounded by the diameter and semicircumference.

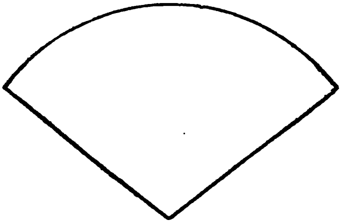
Quarter of a circle is a *Quadrant.* A quadrant is bounded by two radii at right angles, and one fourth of of the circumference.

A Quadrant.



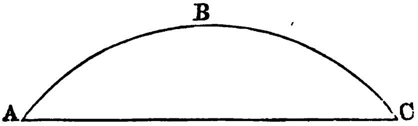
Sectors.





Any part of a circle greater or less than a quadrant, bounded by an arc and two radii, is a *Sector.*

A Segment.

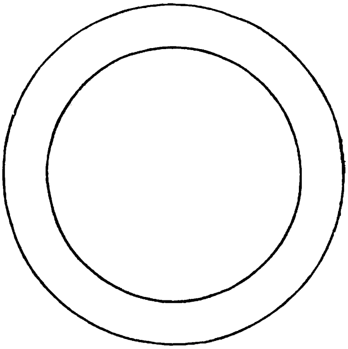


A part of a circle, greater or less than a semicircle, bounded by a chord and an arc, is a *Segment.*

The line A B C is an *Arc.*

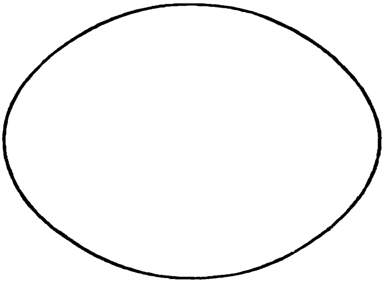
The line A C is a *Chord.*

Concentric Circles.



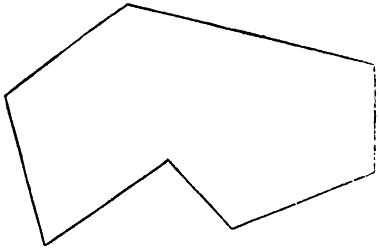
Two or more circles having the same centre on the same plane, are *Concentric Circles.*

An Ellipsis.



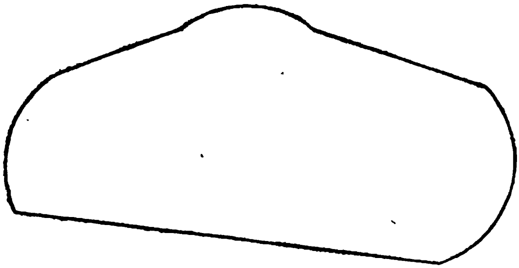
An oval figure, bounded by a single curved line, the opposite points of which, are equally distant from the centre, is called an *Ellipsis.*

An Irregular Figure.



Figures which have unequal sides and angles are *Irregular Figures.*

Compound Figures.



Figures which are bounded by straight and curved lines are called *Compound Figures.*

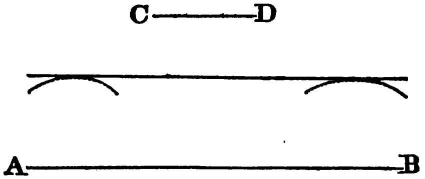
Draw and define a Circle; Circumference: Diameter; Radius; Chord; Arc; Sine; Versed Sine; Tangent; Secant; Semicircle. How is a Semicircle — a Quadrant — a Sector and Segment bounded? Draw Concentric Circles; an Ellipsis; an Irregular Figure; a Compound Figure.

GEOMETRICAL PROBLEMS.

SECTION V.

PROBLEM I.

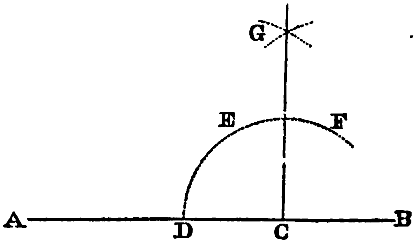
To draw a line parallel to agiven straight line, at a given distance.



At the given distance C D from the line A B describe an arc of a circle; at the same distance from another part of the same line, describe a similar arc; draw a straight line touching these arcs without cutting them, and it will be parallel to A B.

PROBLEM II.

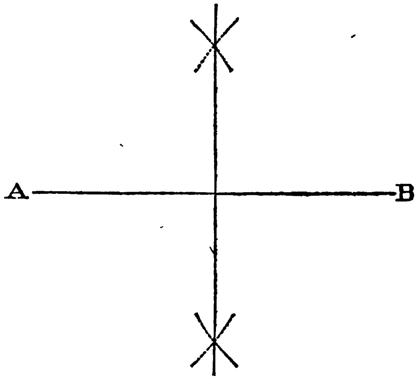
To draw a perpendicular from any part of a given line.



From the point C in the given line A B, describe an arc of a circle greater than a quadrant. From D to F, at the distance of the radius set off a part of the arc from D to E and from E to F; from the points E and F describe intersecting arcs at G; draw a line from C to G and it will be perpendicular to A B.

PROBLEM III.

To bisect or divide a given line into two equal parts.

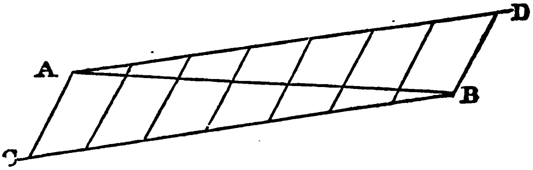


At any distance greater than half of the given line A B with the points A B as centres, describe intersecting arcs above and below the given line; draw a straight line between the intersecting arcs, and it will divide the line A B into two equal parts.

PROBLEM IV.

To divide a given straight line into any number of equal parts.

Divide the line A B into seven equal parts.

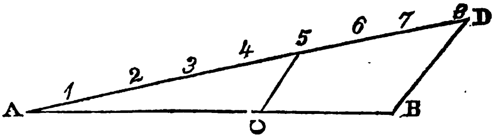


From the points A, B, draw the lines A D and B C, indefinitely, parallel to each other. On them, from A and B set off the number of parts required: join the opposite points, and it will divide the line A B as proposed.

PROBLEM V.

To divide a line in any given proportion.

Divide A B in the proportion of 3 to 5.

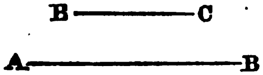


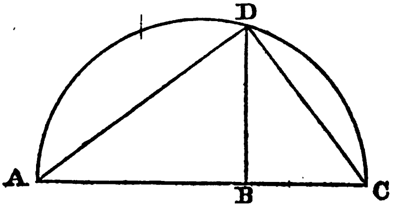
From the point A, draw A D of indefinite length. On it, set off eight equal divisions of indefinite length: join B with the eighth division, and parallel to it join C to the fifth division. Then A B will be divided in the proportion required.

PROBLEM VI.

To find a mean proportional to two given lines.

Find a mean proportional between the lines A B and B C.



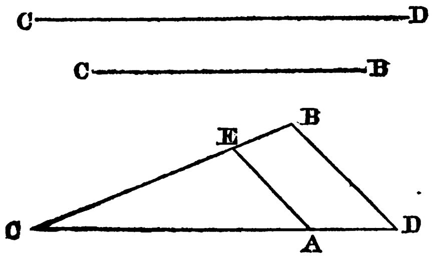


Draw a line A C equal to A B and B C. From the centre of the line A C as diameter, describe a semicircle. At the point B draw a perpendicular B D and it will be a mean proportional between A B and B C.

PROBLEM VII.

To find a third proportional to the less of two given lines.

Find a third proportional to the less of the lines C B C D.

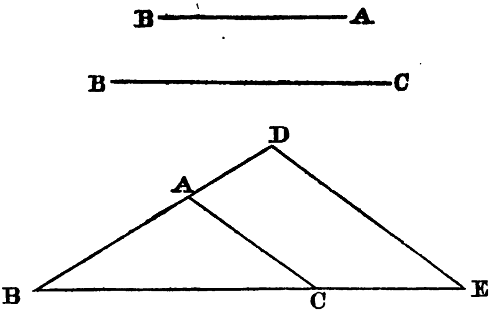


Draw an acute angle having the sides equal to C B, C D; from C D cut off C A equal to C B; join B D and draw A E parallel to it, then C E will be the third less proportional to C D, C B.

PROBLEM VIII.

To find a third proportional to the greater of two given lines.

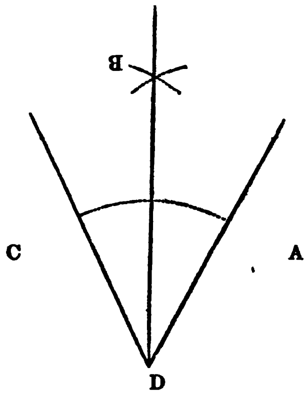
Find a third proportional to the greater of the lines B A, B C.



Draw an acute angle having the sides equal to the lines B A, B C; extend B A to D equal to B C; draw a line from A to C, then draw a line from D parallel to it, and extend B C to E to meet it, then B E will be the proportional required.

PROBLEM IX.

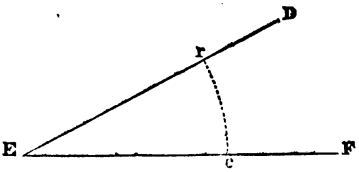
To bisect, or divide a given angle into two equal parts.

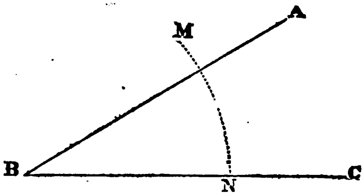


From the vertex B as a centre, describe an arc of a circle within the angle; from the extremities of the arc with the same radius, describe intersecting arcs at D; draw a line from B to D and it will bisect the angle A B C.

PROBLEM X.

At a given point to make an angle equal to a given angle.

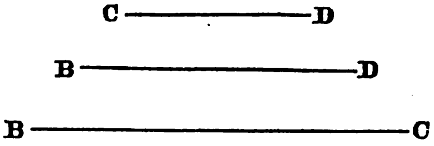


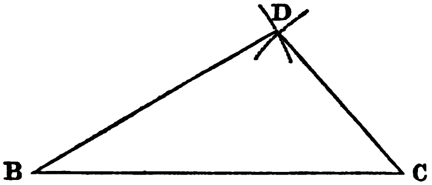


At the point B to draw an angle equal to the angle D E F. From the point B draw B C indefinitely. From E as a centre describe an arc r c, in the angle D E F. With the same radius, and B as a centre, describe an arc M N, from the line B C take the length of the arc r c and set off the same distance M N on the arc M N : draw a line from B through M to A, and it will complete the angle A B C equal to the angle D E F.

PROBLEM XI.

Three sides of a triangle being given to construct it.

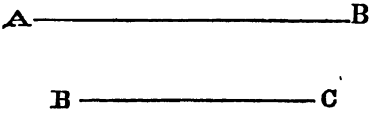


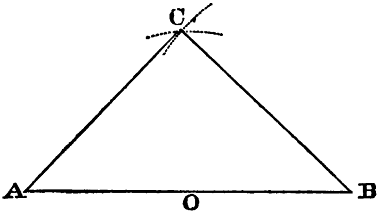


To construct a triangle with the lines B C, B D, C D. Draw B C. With B as a centre, and radius B D, describe an arc; with C as a centre, and radius C D, intersect the arc at D, from D draw lines to B and C, and they form the triangle required.

PROBLEM XII.

The hypothenuse and one side of a right triangle being given to construct it.



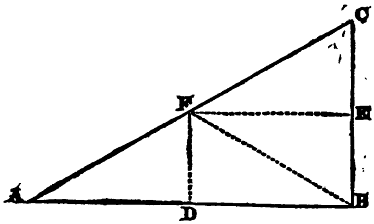


Bisect the hypothenuse A B. With O as a centre and radius A O describe an arc of a circle: with A as a centre and radius B C intersect the arc at C, unite A with C and the triangle will be constructed.

PROBLEM XIII.

To divide a right triangle into four similar and equal triangles.

Divide the triangle A B C into four equal and similar triangles.

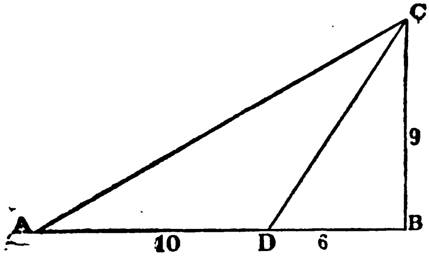


From the centre of the hypothenuse draw the lines F D, F E, perpendicular to the base and altitude and also F B to the opposite angle. This divides the triangle into four similar and equal triangles.

PROBLEM XIV.

To divide a right triangle in any given proportion.

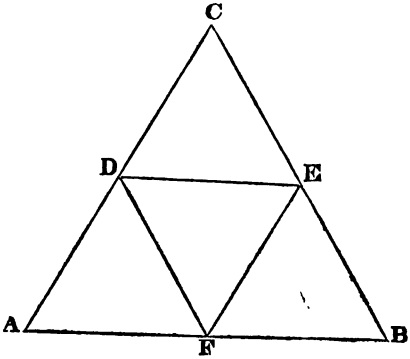
Divide the triangle A B C in the proportion of six to five.



Divide the base into sixteen equal parts, and let A D be ten parts and D B parts, six from D the point of division draw D C and it will divide the triangle in the proportion required.

PROBLEM XV.

To divide an equilateral triangle into four similar and equal triangles.



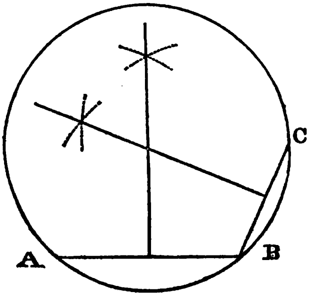
Divide the equilateral triangle A B C into four similar and equal triangles.

Bisect each side of the given triangle and draw lines to the opposite sides. Then will the triangle be divided as required.

PROBLEM XVI.

To describe a circle, the circumference of which shall pass through any three given points not in a straight line.

Draw a circle, the circumference of which shall cut the points A B C.

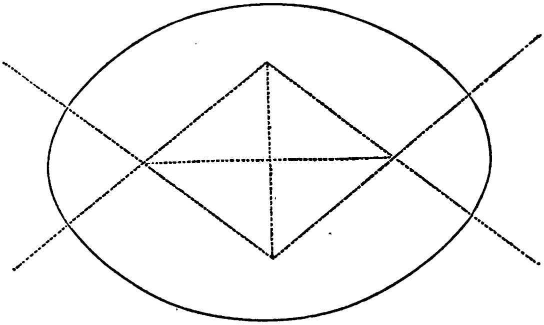


Draw straight lines from A to B and from B to C, bisect these lines with perpendiculars intersecting each other; the point of intersection will be the centre of the circle.

By this problem, it will be easy to find the centre of any circle. If an arc of a circle be given, the whole circle may be found by the same problem.

PROBLEM XVII.

To construct an Ellipsis.



Draw a square or rhombus, so that one of the diagonals shall be vertical, the other horizontal. Extend the sides opposite the vertical diagonal indefinitely: with the vertices as centres, draw four arcs meeting at the extended sides, and they will form the Ellipsis required. The form of the Ellipsis may be varied in any proportion, by varying the proportion of the diagonals of the rhombus.

The above will answer for most practical purposes; but a true ellipsis is generated from a conic section, and has no part of a circle in its composition.

GEOMETRY.

PART THIRD.

CHAPTER I.

MENSURATION OF LINES AND SURFACES.

SECTION I.

Lineal Measure.

6 Points	are	1 Line.
12 Lines	"	1 Inch.
12 Inches	"	1 Foot.
3 Feet	"	1 Yard.
5½ Yards	"	1 Rod.
40 Rods	"	1 Furlong.
8 Furlongs.	"	1 Mile.
3 Miles	"	1 League.
60 Geographical, or 69½ Statute Miles	"	1 Degree on the Earth.

360 Degrees, the circumference of the Earth.

Surveyor's Measure.

7.92 Inches . . . . . are . . . . . 1 Link.
25 Links . . . . . " . . . . 1 Rod.
4 Rods . . . . . . " . . . . . 1 Chain.
10 Chains . . . . . " . . . . 1 Furlong.

In Measuring the Depth of Water:

6 Feet . . . are . . . 1 Fathom.

1. In 2 lines how many points? ANS. 12 points.
2. In 4 inches how many lines? ANS. 48 lines.
3. How many points in one inch? ANS. 72 points.
4. How many points in three lines? ANS. 18 points.
5. In 2½ inches how many lines? ANS. 30 lines.
6. In 3½ lines how many points? ANS. 21 points.
7. How many inches in three feet? ANS. 36 inches.
8. How many inches in 3½ feet? ANS. 42 inches.
9. How many feet in 5 yards? ANS. 15 feet.
10. How many feet in 2½ yards? ANS. 7½ feet.
11. How many feet in 5½ yards? ANS. 16½ feet.
12. How many inches in one yard? ANS. 36 inches.
13. How many yards in two rods? ANS. 11 yards.
14. How many feet in 1 rod? ANS. 16½ feet.
15. In 5 rods how many feet? how many yards? ANS. 82½ feet. 27½ yards.

16. In 15 rods how many yards? Ans. $82\frac{1}{2}$ yards.

17. In 1 yard, 2 feet, 9 inches, how many inches? Ans. 69 inches.

18. In 3 rods, 4 yards, 1 foot, 6 inches, how many inches? Ans. 756 inches.

19. What part of 1 line is 1 point? Ans. $\frac{1}{6}$.

20. What part 1 inch is 1 line? Ans. $\frac{1}{12}$.

21. What part of 1 foot is one inch? 1 line? 1 point?
Ans. 1 inch=$\frac{1}{12}$. 1 line=$\frac{1}{144}$. 1 point=$\frac{1}{864}$.

22. What part of 1 foot 5 lines 10 points, is 8 lines 3 points? Ans. $\frac{51}{904}$.

23. What part of 1 yard is 2 feet? Ans. $\frac{2}{3}$.

24. 1 foot is what part of 3 yards? Ans. $\frac{1}{9}$.

25. 1 inch is what part of 1 yard? Ans. $\frac{1}{36}$.

26. 1 foot 9 inches is what part of 1 yard 2 feet? Ans. $\frac{7}{20}$.

27. In 11 yards how many rods? Ans. 2 rods.

28. What part of 1 rod is 1 yard? Ans. $\frac{2}{11}$.

29. What part of 1 rod 3 yards, is 5 yards? Ans. $\frac{10}{17}$.

30. What part of 1 rod is 1 foot? Ans. $\frac{2}{33}$.

31. 1 foot 6 inches, is what part of 1 yard 1 foot? Ans. $\frac{3}{8}$.

32. What part of 1 yard is 6 inches? Ans. $\frac{1}{6}$.

33. In 1 rod how many inches? Ans. 198 inches.

34. What part of 1 rod 3 feet is $5\frac{1}{2}$ feet? Ans. $\frac{11}{39}$.

35. What part of $7\frac{3}{4}$ rods is $5\frac{1}{2}$ yards? Ans. $\frac{4}{31}$.

36. A surveying wheel turns twice in going 1 rod; how many times will it turn in going $577\frac{1}{2}$ feet? Ans. 35 times.

37 How many rods in 4 furlongs? Ans. 160 rods.

38. How many feet in 1 furlong? Ans. 660. feet.

39. In 3 furlongs 15 rods, 8 feet, how many feet? Ans. $2235\frac{1}{2}$. feet.

40. In 1 furlong how many inches? Ans. 7920 inches.

41. In 2 furlongs, 10 rods, 12 feet, 6 inches, how many inches? Ans. 17970 inches.

42. In six miles how many furlongs? Ans. 48 furlongs.

43. In one mile how many rods? Ans. 320 rods.

44. How many yards in 1 mile? Ans. 1760 yards.

45. In 1 mile how many feet? Ans. 5280 feet.

46. In 1 mile how many inches? Ans. .63360 inches.

47. In $2\frac{2}{3}$ leagues how many miles? Ans. 8 miles.

48. How many rods in 1 league? Ans. 960 rods.

49. How many yards in 1 league, 1 rod, and 1 yard? Ans. $5286\frac{1}{2}$ yards.

50. How many rods in 135 feet? Ans. $8\frac{2}{11}$ rods.
Change 135 feet to half feet, and divide by 33, the number of half feet in a rod.

51. A Surveyor's chain is divided into 100 links, each link is 7.92 inches. How many feet long is the chain? Ans. 66 ft.

52. 25 links are 1 rod. How many feet are 1 rod? Ans. $16\frac{1}{2}$ ft.

53. 4 rods is the length of the chain. How many feet long is it? Ans. 66 ft.

54. How many inches long is the chain? Ans. 792 inches.

55. There are 100 links in the chain, what is the length of 1 link? Ans. 7.92 in.

56. How many rods are 4½ chains?
ANS. 18 rods.

57. How many rods are there in 10 chains and 25 links?
ANS. 41 rods.

58. How many feet long is a piece of land that measures 10 chains, 1 rod in length?
ANS. 676½ feet.

59. The distance from Boston to Roxbury is three miles how many rods is it? ANS. 960 rods.

60. The Middlesex Canal is 30 miles in length, how many chains are equal to the length of it?
ANS. 2400 chains.

61. What is the length of the Middlesex Canal in feet?
ANS. 158400 ft.

SECTION II.

Mensuration of Surfaces.

A Surface is a space, the boundaries of which are lines.

A Surface has two dimensions only — length and breadth.

Surfaces are either plane or curved.

A plane surface may be represented by one side of a smooth sheet of paper, lying flat upon a table.

A curved surface may be represented by one side of a smooth sheet of paper rolled up so as to unite the opposite edges.

By superficial measure we determine how many squares of equal dimensions are contained on any given surface.

These squares are of known dimensions, being one line, one inch, one foot, one yard, one rod, &c. on each side, called one square line, one square inch, square foot, square yard, &c.

The measure of a surface is called its Area.

Tables of Square Measure.

36	points . . .	are . .	1 square line.
144	square lines .	" . .	1 square inch.
144	square inches .	" . .	1 square foot.
9	square feet . .	" . .	1 square yard.
30¼	square yards .	" . .	1 square rod.
40	square rods .	" . .	1 rood.
4	roods . . .	" . .	1 acre.
640	acres . . .	" . .	1 sq. mile.

Surveyor's Square Measure.

62.7264	square inches	are	1 square link.
625	square links	"	1 square rod.
16	square rods	"	1 square chain.
100	square chains	"	1 square furlong.
64	square furlongs	"	1 square mile.
9	square miles	"	1 square league.

Parallelograms.

To find the area of a parallelogram.

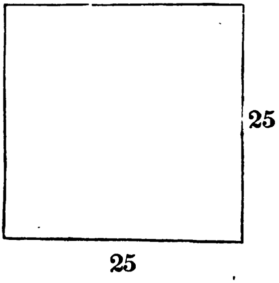
Multiply the base by the altitude.

The *base* is the side upon which the figure stands.

The *altitude* is the perpendicular height.

SECTION III.

1. What is the area of a square, the side of which is 25?



25 × 25 = 625 ANS. 625.

2. Each side of a square foot is 12 inches, how many square inches in a square foot? ANS. 144 sq. inches

3. What is the area of a square, each side of which is 5 feet? ANS. 25 sq. feet.

4. What is the area of a square each side of which is 15 inches? ANS. 225 sq. in.

5. How many feet in a square yard? ANS. 9 sq. feet.

6. How many square inches on a table, each side of which is three feet? **Ans.** 1296 sq. in.

7. If the side of a square piece of land be one rod, how many feet is the area? **Ans.** 272$\frac{1}{4}$ sq. ft.

8. What is the area of a square each side of which is 5 feet 6 inches? **Ans.** 30$\frac{1}{4}$ sq. ft.

9. How many square rods are there in an acre? **Ans.** 160 sq. rods.

10. How many acres in a piece of land each side of which measures one mile?

Note. 320 rods in length are 1 mile.

Divide the number of square rods in a square mile by the number in an acre. **Ans.** 640 acres.

11. If the side of a square field be forty rods, how many acres does it contain? **Ans.** 10 acres.

12. How many square feet in a board 24 inches on each side? **Ans.** 4 sq. ft.

13. How square yards of pavement in a court 25 feet square? **Ans.** 69$\frac{4}{9}$ yds.

14. What is the area of a room 13 feet 9 inches square? **Ans.** 189 sq. ft. 9 sq. in.

15. What is the area of a piece of land 35.75 feet on each side? **Ans.** 1278.0625.

16. A farmer has three fields, the first contains twice as much as the second, the second measures 16 rods on each side, and the third contains as much as the first and second, how many acres do the three contain? **Ans.** 9 acres, 2 roods, 16 rods.

17. How many square feet on the floor of a room 15 feet 9 inches square?

Questions like the last may be performed by decimals or duodecimals. Both methods will be given.

By Decimals.	By Duodecimals.
9 inches are $\frac{9}{12}$ of a foot = .75	
15.75	15 " 9
15.75	15 " 9
7875	11 " 9 " 9
11025	236 " 3
7875	
1575	Sq. feet 248 " 0 " 9 sq. in.
Sq. ft. 248.0625	

SECTION IV.

Application of Square Root.

To find the side of a given square.

The square root of the given square is the side required.

1. What is the length of the side of a square foot?

144 square inches are 1 square foot. The square root of 144 is 12. Ans. 12 inches.

2. What is the length of each side of a square, the area of which is 81 square feet? Ans. 9 feet.

3. What is the length of the side of a square containing 225 square feet? Ans. 15 feet.

4. What is the side of a square containing 1 rod?

Ans. 55 square yds.

5. What is the side of a square room containing 420¼ feet? Ans. 20½ feet.

6. What is the length of a side of a square field containing 9025 square rods? Ans. 95 rods.

To find the side of a square which shall be equal in area to any given number of feet, inches, &c.

The square root of the given area will be the side of a square, equal in area.

1. What will be the side of a square to contain 225 inches?

The square root of 225 is 15. Ans. 15 inches.

2. What will be the side of a square equal in area to a circle containing 625 inches? Ans. 25 inches.

3. How many marble tiles, each nine inches square, are contained on one side of a square court, the area of which is 225 square feet? Ans. 20 tiles.

4. What is the side of a square equal in area to a circle containing 2025 square inches? Ans. 45.

5. A gentleman wishes to exchange a piece of ground in the form of a crescent, containing 1024 square feet, for a piece in the form of a square of equal area, what will be the side of the square?

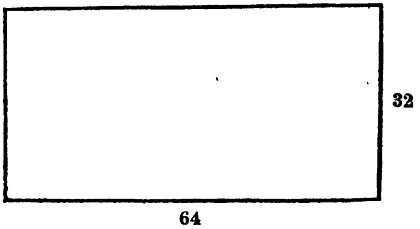
The square root of 1024 is 32. Ans. 32 feet.

6. What will be the side of a square field which shall contain as much land as another field which is 48 rods long and 12 rods wide? Ans. 24 rods.

SECTION V.

Oblong.

1. What is the area of an oblong, the base of which is 64 and altitude 32?



$64 \times 32 = 2048$ Ans. 2048.

2. How many square inches in an oblong 16 inches long and nine inches wide? Ans. 144 square inches.

3. What is the area of a parallelogram 18 inches in length and 12 inches in width?

Ans. 1 square foot and 72 square in. $= 1\frac{1}{2}$ sq. ft.

4. How many square feet in a room 19 feet long and 15 feet wide? Ans. 285 sq. ft.

5. What is the superficial measure of a board 13 feet 6 inches long and 21 inches wide? Ans. $23\frac{5}{8}$ sq. ft.

6. What is the area of a piece of land 44 feet long and 27 feet 8 inches wide? Ans. $1217\frac{1}{3}$ sq. ft.

7. If an oblong measure be 5 feet $7\frac{1}{2}$ inches in length and 3 feet $5\frac{1}{4}$ inches in width, what is the area?

The answer is $19\ \frac{4}{12} + \frac{4}{1728} + \frac{6}{20736}$ sq. feet.

$\frac{4}{12} = \frac{1}{3}$ of a foot. Ans. $19\frac{1}{3}+$ sq. ft.

8. What is the number of square feet on a floor 22 ft. 6 inches by 13 feet 9 inches. Ans. $309\frac{3}{8}$ sq. ft.

9. How many square feet of boards will cover the floor of a hall $50\frac{3}{4}$ feet long, and 36 feet wide? Ans. 1827.

10. How many yards of carpeting 1 yd. wide will cover the above floor? Ans. 203 yds.

11. How many yards of carpeting $1\frac{1}{2}$ yds. wide will cover the floor of a room 20 feet long, $16\frac{7}{8}$ feet wide? Ans. 25 yds.

12. What is the surface of an oblong 3 feet 6 inches by 2 feet 10 inches? Ans. $9\frac{11}{12}$ sq. ft.

13. How far from the end must a board be cut that is 18 inches wide, so that the piece may measure 1 square foot? Ans. 8 inches.

14. What length of board 10 inches wide will be equal to one square foot? Ans. $14\frac{2}{5}$ in.

15. How many inches wide must a board be that is 16 inches long, to make a square foot? Ans. 9 in.

16. How many square feet and inches does a board measure that is 13 feet 6 inches long and 2 feet wide? Ans. 27 sq. ft. and 48 sq. in.

17. How many square feet of boards will it require to lay a floor 17 feet 9 inches long and 15 feet 6 inches wide? Ans. $275\frac{1}{8}$ sq. ft.

18. How many feet in a board 13 feet 6 inches long and 15 inches wide? Ans. $16\frac{7}{8}$ ft.

19. How many square feet in a square rod? Ans. $272\frac{1}{4}$ sq. feet.

20. There are forty square rods in a rood, how many square feet are there in a rood? Ans. 10890 sq. ft.

21 How much in length of land that is 5 rods wide, will it require to make an acre?

Note. 160 square rods make an acre.

Ans. 32 rods.

22. How many square feet are there in 1 acre? Ans. 43560 sq. ft.

23. St. Peter's Church at Rome is 720 feet long and 510 feet broad, how many acres of land does it cover.

ANS. 8 acres 1 rood 28 rods 207 sq. ft.

24. What will be the cost of a marble slab 6 feet 6 inches long, and 2 feet 4 inches wide at 75 cts. per square foot?

ANS. $\$11.37\frac{1}{2}$ cts.

25. What will it cost to paint a floor 13 feet 6 inches long, and 9 feet 6 inches wide, at 9 cts. per sq. yd?

ANS. $\$1\ 28\frac{1}{4}$ cts.

Painting is measured by the square yard. 9 sq. feet are 1 sq. yard.

26. How much will it cost to paint a wall 140 feet long and 6 feet high, at 15 cts. per square yard?

ANS. $14.

27. How many square feet are there in a floor 15 feet $7\frac{3}{4}$ inches in length and 12 feet $6\frac{2}{3}$ inches in breadth?

ANS. $196\frac{7}{16}$ feet.

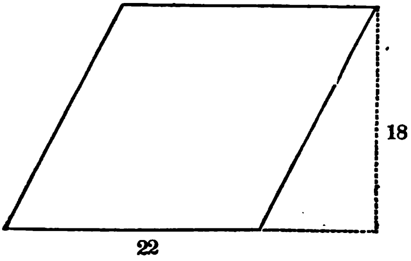
28. How many square feet of boards will cover the floor of a hall 60 feet $8\frac{1}{4}$ inches in length and 35 feet $10\frac{3}{4}$ inches in breadth?

ANS. 2178 sq. ft. $61\frac{11}{16}$ sq. in.

SECTION VI.

Rhombus.

1. What is the area of a rhombus, the base of which is 22 and altitude 18?



The altitude is the perpendicular height.

$22\times18=396$ for the area.

2. What is the area of a rhombus, each side of which is 12 feet 6 inches, and altitude 8 feet 4 inches?

Ans. $106\frac{1}{6}$ sq. ft.

3. What is the area of a piece of land, in the form of a rhombus, each side of which measures 40 feet 9 inches and perpendicular distance between the sides 32 ft. 3 in.?

Ans. $1310\frac{41}{48}$ sq. ft.

4. What is the area of a room, each side of which is $15\frac{3}{4}$ feet, and the perpendicular distance between the walls 12 feet 6 inches? Ans. $196\frac{7}{8}$ sq. ft.

5. What is the area of a field in the form of a rhombus, each side measuring 18 rods 10 feet, and perpendicular distance between the sides 14 rods 4 yards?

Ans. 274 sq. rods 50 sq. yds.

Land is measured by a surveyor's chain four rods in length, each rod twenty-five links, each link being 7.92 inches.

6. How many acres are there in a field, each side measuring 10 chains, and perpendicular distance 8 chains?

Ans. 8 acres.

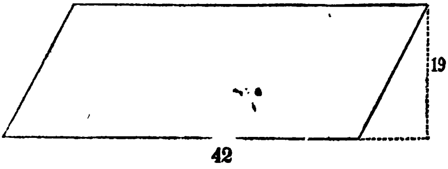
7. Required the area of a piece of land in the form of a rhombus, each side of which measures 725 feet, and perpendicular distance between the sides 635 feet, how many acres does it contain?

Ans. 10 acres 91 sq. rods.

SECTION VII.

Rhomboid.

1. If the base of a Rhomboid be 42 and the altitude 19, what is the area?



42×19=798 the area.

2. If the two sides of a rhomboid be each 8 inches, the other two each 5 inches, and altitude 3 inches, what is the area?

Ans. 24 sq. inches.

3. How many square yards of painting are there on a floor 28 feet 6 inches in length, and 18 feet perpendicular width?

Ans. 57 sq. yds.

4. A piece of land in the form of a rhomboid measures 640 links on the two longest sides, and is 144 links perpendicular width, how many square rods does it contain ?

Ans. $147\frac{57}{125}$ sq. rods.

SECTION VIII.

To form a parallelogram of given area, whose length shall bear a given proportion to its width.

Divide the area of the parallelogram by the given proportion, and the square root of the quotient will be the width required.

Multiply the width by the proportion, and the product will give the length.

1. What will be the dimensions of a parallelogram containing 243 square inches, the length to be three times the width.

$243 \div 3 = 81$, the square root of 81 is 9, the width.

$9 \times 3 = 27$ the length.

Ans. 9 inches wide, 27 inches long.

2. It is required to cut a piece of marble, the surface of which is to be 9 square feet, so that the length shall be in proportion to the width as 4 to 1, what will be the dimensions? Ans. length ~~24~~ 72 inches, width ~~36~~ 18 inches.

3. A farmer wishes to lay out a piece of land to contain 45 acres, so that the length shall be equal to twice the breadth. Required the dimensions.

Change the acres to square rods.

Ans. 120 rods long, 60 rods wide.

4. An army of 14700 men is to be formed into a solid column, so that there shall be only one third as many men in rank as there are in file ; how shall the column be formed ? Ans. 70 men in rank, 210 men in file.

SECTION IX.

To find the superficial contents of a board or any plain figure of unequal width, the ends or bases parallel.

Multiply half the sum of the two ends by the length, the product will be the area.

1. What is the measure of a board 13 feet long, the ends being 15 inches and 9 inches?

$15+9=24$ half of $24=12$; 12 inches are 1 foot.

$13\times1=13$ Ans. 13 sq. ft.

2. What is the area of the floor of a hall 40 feet long, 25 feet wide at one end, and 20 feet wide at the other?

Ans. 900 sq. ft.

3. How many square feet are there in a farm, the two parallel sides of which are 75 feet and 95 feet, and the perpendicular distance between them 150 feet?

$75+95=170$ half of this sum is 85.

$85\times150=12750$ Ans. 12750 sq. ft.

SECTION X.

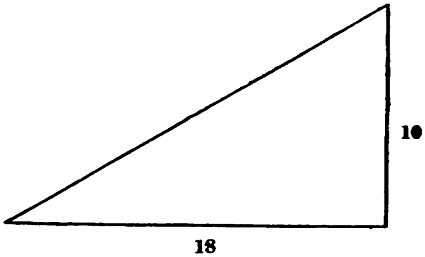
Triangles.

To find the area of a Triangle.

Multiply the base by half the altitude; the product is the area.

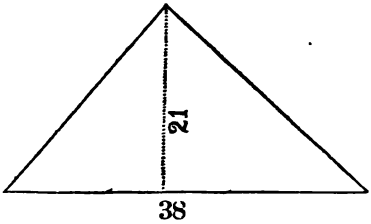
Every triangle is equal to half the area of its circumscribing parallelogram.

1. What is the area of a right triangle, the base of which is 18 and perpendicular 10?



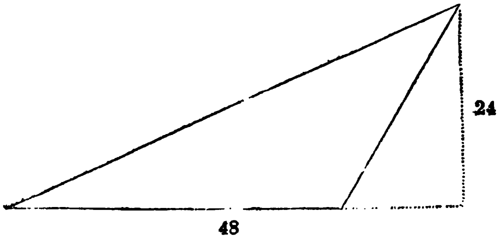
Half of 10 is 5; 18×5=90 the area of the triangle.

2. What is the area of an acute triangle, the base of which is 38, and altitude 21?



Half of 21=10.5; 38×10.5=399 the area.

3. What is the area of an obtuse triangle, the base of which is 48, and altitude 24? Ans. 576.



1. What is the area of a right triangle the base of which is 36 inches, and perpendicular 20 inches?
ANS. 360 sq. inches.

2. What is the area of a right triangle, the sides of which are 27, 36, and 45 inches? ANS. 486 sq. in.

In a right triangle the longest side is the hypothenuse. Either of the other sides is a perpendicular.

3. If the base of a triangle be 99 inches and altitude 88 inches, how many square feet is its surface?
ANS. 3¼ sq. ft.

4. A triangular piece of marble measures 3 feet 3½ inches in length and 17 inches greatest width, what is the area? ANS. 2 sq. ft. 47¾ sq. in.

5. The base of a triangle is 28.4 yards, and the altitude 18.4 yards, what is the area? ANS. 261.28 sq. yds.

6. What is the area of an equilateral triangle, each side of which is 19 inches, and altitude 16.454 inches?
ANS. 78.1565 sq. in.

7. What is the area of a triangle, the base of which is 18 feet 4 inches, and altitude 11 feet 10 inches?
ANS. 107⅝ sq. in.

8. What is the area of a right triangle, the sides of which are 15, 20, and 25 feet? ANS. 150 sq. ft.

9. The sides of a garden in the form of a right triangle are 225, 300, and 375 feet, what is the area?
ANS. 33750 sq. feet.

10. If two sides of a triangle are each 18 inches, the altitude 17 inches, and base 12 inches, what kind of triangle is it? what is its area?
ANS. Isosceles. 102 sq. inches.

11. There is a piece of land in the form of a right triangle, two sides of which are equal, but on account of ob-

stacles cannot be measured, but the side opposite the right angle measures 90 rods. What is the area?

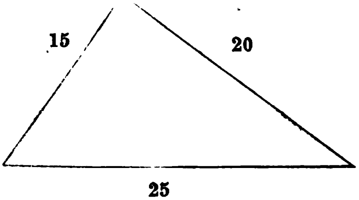
Ans. 12 acres, 2 roods, 25 sq. rods.

Note. If a right triangle be isosceles, the area will be equal to one fourth the square of the hyothenuse.

Having the three sides of a triangle given, to find the area without the aid of the perpendicular.

From half the perimeter, subtract each side; from the product of the perimeter multiplied by the three differences, extract the square root, and it will be the area of the triangle.

1. What is the area of a triangle, the sides of which are 15, 20, and 25 inches?



$25 + 20 + 15 = 60$ the perimeter

half of which is $30 - 25 = 5$

$30 - 20 = 10$

$30 - 15 = 15$

$30 \times 10 \times 15 \times 5 = 12500$, the square root of which is 150, the area. Ans. 150 sq. in.

2. What is the area of a triangle, the sides of which are 75, 100, and 125? Ans. 3750.

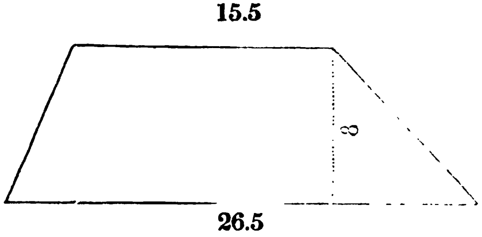
SECTION XI.

Trapezoid.

To find the area of a Trapezoid.

Multiply half the sum of the two parallel sides by the altitude, the product is the area.

1. What is the area of a trapezoid the parallel sides of which are 26.5 and 15.5, and the altitude 8 ?



$26.5+15.5=42$; half of which is $21\times8=168$ area.

2. A marble slab is 16 inches and 24 inches on the two parallel sides, and 12 inches wide, what is the area ?
Ans. 240 sq. inches.

3. If one of the parallel sides of a trapezoid be 6 feet, the other 8 feet, and altitude 4½ feet, what is the area ?
Ans. 31½ sq. in.

4. If two opposite sides of a garden are 35 feet and 25 feet, and perpendidular width 15 feet, what is the area ?
Ans. 450 sq. ft.

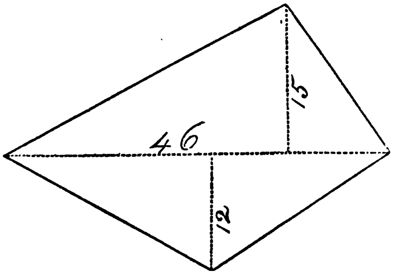
SECTION XII.

Trapezium.

To find the area of a Trapezium.

Multiply half the diagonal by the sum of the two perpendiculars, the product is the area.

1. What is the area of a trapezium, the diagonal of which is 46, the perpendiculars 15 and 12?



Half of 46 is 23 $15+12=27$

$23\times27=621$ the area.

2. If the diagonal of a trapezium be 8 feet, and the perpendiculars 24 and 30 inches, what is the area?

ANS. 18 sq. ft.

3. What is the area of a trapezium, the diagonal of which is 34 feet 9 inches, and the two perpendiculars 19 feet and 12 feet 6 inches? ANS. $548\frac{7}{48}$ sq. ft.

4. The perpendiculars of a piece of land are 178 and 183 feet, and the diagonal is 225 ft.; what is the area?

ANS. 40612.5 sq. feet.

5. In a field in the form of a trapezium the diagonal is 85 feet and the perpendiculars 32 and 35 feet, what number of square feet does it contain?

ANS. $2847\frac{1}{2}$ sq. ft.

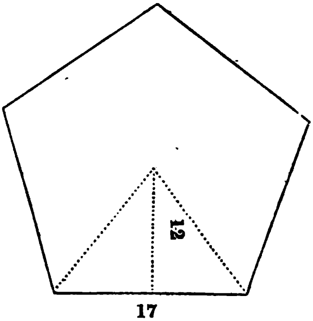
SECTION XIII.

Regular Polygons.

To find the area of any regular Polygon.

Multiply half the perimeter or sum of all the sides by the perpendicular distance from the centre to one of the sides. The product is the area.

1. What is the area of a regular pentagon, each side of which is 17, and perpendicular 12?



17 × 5 = 85, half of which is 42.5, this multiplied by the altitude 12 = 510 for the area.

This divides the pentagon into five isosceles triangles, the base of each of which is 17 and altitude 12.

In the same manner may be found the area of a hexagon, heptagon, or of any regular polygon of any number of sides.

The area of a circle may be found in the same manner.

2. What is the area of a piece of marble in the form of a hexagon, whose side is 12 inches, and perpendicular 10.39 inches? Ans. 374.04 sq. in.

3. Each side of a regular pentagon is 10 inches, and perpendicular 6.8919. What is the area?

Ans. 172.047.

4. What is the area of a regular hexagon, each side of which is 15 inches, and perpendicular 22.99 inches?

Ans. 584.55

5. The top of a marble table, of an octangular form, measures 16 feet perimeter. What will it cost at 1¼ dolls. per square foot? Ans. $21.7279.

SECTION XIV.

The Circle.

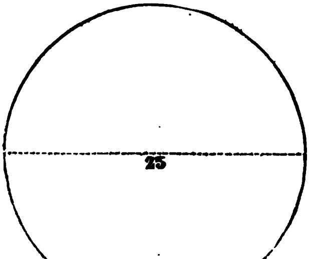
A circle is the most capacious of all plane figures.

It contains a larger area within the same perimeter, and combines a greater variety of useful properties than any other geometrical figure.

To find the area of a circle having the diameter given.

Multiply the square of the diameter by .7854.

1. What is the area of a circle, the diameter of which is 25?



25×25=625×.7854=490.875 the area.

If the circumference only be given, multiply the square of the circumference by .07958.

2. What is the area of a circle, the diameter of which is 10 inches? Ans. 78.54 sq. in.

3. What is the area of a circular pond 25 rods in circumference? Ans. 49.7375 sq. rods.

4. What is the area of a circle, whose radius is 4 feet? Ans. 50.2656 sq. ft.

5. If the circumference of a circle be 15 feet, what is the area? Ans. 17.9 sq. ft.

6. If the diameter of a circle be 144 inches, what is the area? Ans. 108.56 sq. ft.

7. What is the area of a circular garden, the circumference of which is 94.248 feet, and diameter 30 feet? Ans. 706.86 sq. ft.

8. A surveyor's wheel is made to turn twice in going 1 rod. What is the area of a field around which it turns 300 times? Ans. 11 acres, 30.55 sq. rods.

9. A circle has a diameter of 4 feet. What is the radius of a circle four times as large? Ans. 4 feet.

Circles are in proportion to each other as the squares of their radii, or diameters.

The square of 4 is 16, four times as large would be 64, the square root of this is 8 the diameter, and radius 4.

If the diameter of a circle be 1, the circumference will be 3.1416. And the area will be .7854 equal to one fourth of the circumference.

If the diameter be 2 the area $=\frac{1}{2}$ the circumference.
" " 3 " $=\frac{3}{4}$ "
" " 4 " = the circumference.
" " 5 " $=\frac{5}{4}$ "
" " 6 " $=\frac{3}{2}$ "
" " 7 " $=\frac{7}{4}$ "
" " 8 " =2 times "
" " 9 " $=2\frac{1}{4}$ "
" " 10 " $=2\frac{1}{2}$ "
" " 11 " $=2\frac{3}{4}$ "
" " 12 " =3 times "
" " 13 " $=3\frac{1}{4}$ "
" " 14 " $=3\frac{1}{2}$ "
" " 15 " $=3\frac{3}{4}$ "
" " 16 " =4 times "
" " 17 " $=4\frac{1}{4}$ "
" " 18 " $=4\frac{1}{2}$ "
" " 19 " $=4\frac{3}{4}$ "
" " 20 " =5 times "

Therefore, dividing the diameter of any circle by 4, and multiplying the circumference by the quotient, gives the area of the circle.

The area of a circle is equal to the curved surface of a cylinder, whose base and altitude are equal to the radius of the circle.

The exact proportion of the diameter to the circumference of a circle has never been discovered. The solution of this problem has exercised the ingenuity of mathematicians for ages. The perfect accuracy of important principles in science depends upon its discovery. Immense premiums have been offered for its solution, in Russia, Germany, France, and Great Britain. But it remains for some gifted mind of a future age to enjoy the honor of knowing that which now belongs only to the "Great Geometer" of the Universe, and the reward for communicating it to the world.

Although the exact ratio of the diameter to the circumference has not been discovered, yet the approximate ratio which has been found is sufficiently accurate for all practical purposes. Archimedes, more than two thousand years ago, found the ratio to be nearly as 1 to 3⅐.

Franciscus Vieta, by means of inscribed and circumscribed polygons, carried the ratio to ten places of decimals; finding that if the diameter be 1, the circumference will be 3.1415926535.

Nearer ratios have, from time to time, been assigned by successive mathematicians; so that the last and most accurate ratio of the diameter to the circumference, is carried to the extent of five hundred and fifty places of decimals.

If the diameter of a circle be 1,
The circumference will be 3.1416.

This proportion is sufficiently correct for common purposes.

It varies less than one one-hundred-thousandth part of an unit.

Having the diameter given, to find the circumference.

Multiply four times the diameter by .7854, the product will be the circumference.

1. If the diameter of a circle be 5, what will be the circumference?

$4\times5=20\times.7854=15.708$ the circumference.

2. If the diameter of a circle be 10 inches, what will be the circumference? Ans. 31.416 inches.

3. What is the circumference of a circle, the diameter of which is 24 inches? ANS. 6.2832 feet.

Having the circumference given to find the diameter.

Divide one fourth of the circumference by .7854, the quotient is the diameter.

1. If the circumference of a circle be 15.708, what will be the diameter?

$\frac{1}{4}$ of $15.708 \div .7854 = 5$ the diameter.

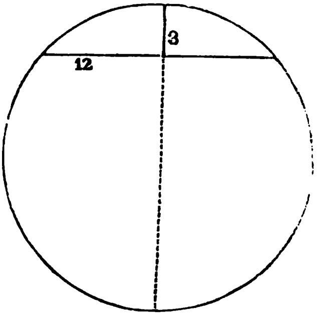
2. If the circumference of a circle be 12.5664 feet, what will be the diameter? ANS. 4 feet.

3. What is the diameter of a circle, the circumference of which is 31.416 inches? ANS. 10 inches.

The chord and versed sine of a circle given to find the diameter.

Divide the square of half the chord by the versed sine, and to the quotient add the versed sine, the sum will be the diameter.

If the chord of an arc of a circle be 12, and versed sine 3, what will be the diameter?



Half the chord is 6, the square of 6 is 36;

$36 \div 3 = 12 + 3 = 15$ the diameter.

2. If the chord of an arc be 16 and versed sine 4, what will be the diameter? Ans. 20.

3. If the chord be 24 and versed sine 8, what will be the diameter? Ans. 26.

4. What will be the diameter of a circle, the chord of which is 54 inches, and versed sine 9 inches?

Ans. 90 inches.

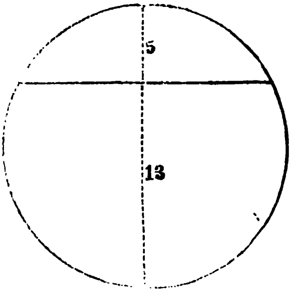
5. What is the diameter of a circle, the chord of which is 72 feet, and versed sine 13.5 feet?

Ans. 109.5 ft.

The diameter and versed sine given, to find the chord.

From the diameter subtract the versed sine, multiply the remainder by the versed sine, and the square root of the product is half the chord.

1. If the diameter of a circle be 18, and versed sine 5, what will be the chord?



18—5=13×5=65 the square root of 65=8.0622, half the chord.

2. What will be the length of a chord of an arc of a circle, the diameter of which is 60 inches, and versed sine 6 inches? Ans. 36 inches.

3. What will be the chord of an arc of a circle, the diameter of which is 90, and the versed sine one tenth of the diameter? Ans. 54 in.

4. If the diameter of a circle be 130, what will be the length of a chord which divides the diameter in the proportion of 5 to 21? Ans. 110.

Having the area of a circle given, to find the diameter.

Divide the area by .7854, and the square root of the quotient will be the diameter.

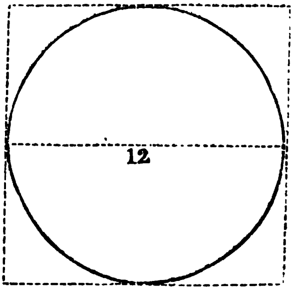
1. What will be the length of the diameter of a circle the area of which is 153.9384 inches? Ans. 14 in.

Having the area of a circle given, to find the circumference.

Divide the area by .07958, the square root of the product will be the circumference.

1. What will be the circumference of a circle, the area of which is 176.715 inches? Ans. 47.123 in.

The square of the diameter of a circle is equal to the area of a circumscribing square.



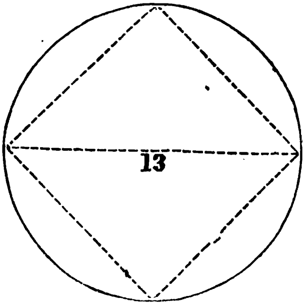
12×12=144, the area of the circumscribing square.

1. If the diameter of a circle be 6 inches, what is the area of a circumscribing square?

$6\times6=36$ Ans. 36 sq. inches.

2. What is the side of a square circumscribing a circle, the radius of which is 4 feet? Ans. 8 feet.

Multiply the diameter of a circle by the radius, the product is the area of an inscribed square.



Diameter 13; radius $6\frac{1}{2}$. $13\times6\frac{1}{2}=84\frac{1}{2}$, the area of the inscribed square.

1. What is the area of a square inscribed in a circle, the diameter of which is 10 inches?

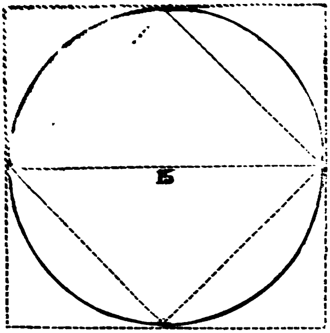
Diameter 10; radius 5. $10\times5=50$.

Ans. 50 sq. feet.

2. If the radius of a circle be 15 feet, what is the area of an inscribed square? Ans. 450 sq. ft.

The area of a circumscribed square is to the area of an inscribed square as 2 to 1.

1. What are the areas of a circumscribed, and of an inscribed square, the diameter of the circle being 15 inches?

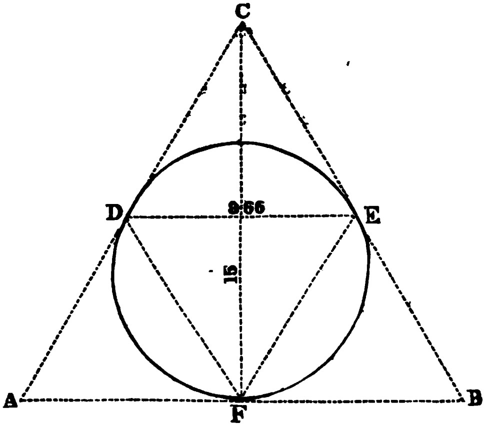


15×15=225, the area of the circumscribed square.
15×7.5=112.5, the area of the inscribed square.
225 is to 112.5 as 2 to 1.

2. If the diameter of a circle be 10 feet, what will be the areas of an inscribed and a circumscribed square?

Inscribed square, 50 sq. in.
Circumscribed square, 100 sq. in.

The area of an inscribed equilateral triangle is to the area of a circumscribed equilateral triangle, as 1 to 4.



17.32×7.5=129.9 area of the circumscribed triangle, A B C.

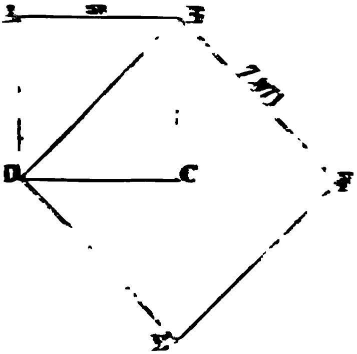
8.66×3.75=32.475 area of the inscribed triangle, D E F.

$\frac{32475}{129900}=\frac{1}{4}$ Therefore, the inscribed triangle is to the circumscribed triangle, as 1 to 4.

SECTION XV.

Miscellaneous.

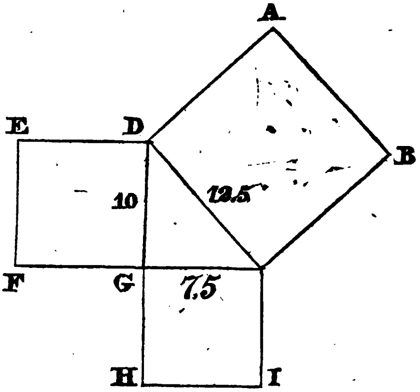
The area of a square formed on the diagonal of a square is equal to twice the area of the square to which the diagonal belongs.



$5 \times 5 = 25$, area of the square A B C D.

$7.071 \times 7.071 = 50$, area of the square B D E F.

The area of a square formed on the hypothenuse of a right triangle, is equal to the sum of the squares of the base and perpendicular.



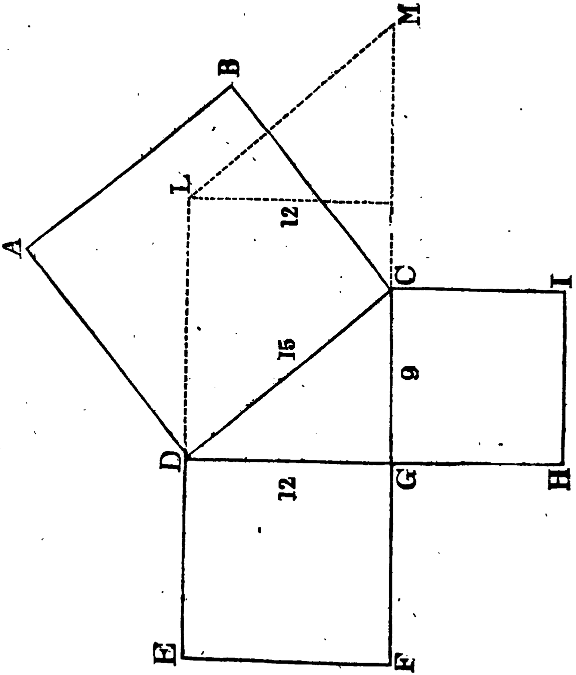
10×10=100, square of theperpendicular.

7.5×7.5=56.25, square of the base.

12.5×12.5=156.25, square of the hypothenuse.

100+56.25=156.25, sum of the squares of the base and perpendicular.

The area of a rhombus formed on the hypothenuse of a right triangle, having the perpendicular of the triangle for its altitude, is a mean proportional between the areas of the hypothenuse and perpendicular.



15×15=225, square of the hypothenuse A B C D.
12×12=144, square of the perpendicular D E F G.
9×9=81, square of the base C G H I.
15×12=180, area of the rhombus D G L M.
180 is a mean proportional between 144 and 225.

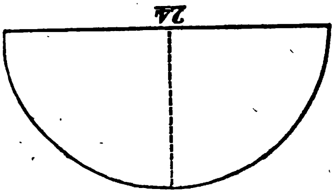
SECTION XVI.

Semicircle.

To find the area of a semicircle.

Multiply the diameter by the radius, and the product by .7854.

1. What is the area of a semicircle, the diameter of which is 24?



24×12=288×.7854=179.0712, the area.

If the semicircumference only be given.

Multiply the square of the semicircumference by .15916, *the product will be the area.*

2. If the diameter of a semicircle be 15 inches, what will be the area? Ans. 88.3575 square in.

3. What is the area of a semicircle, the diameter of which is 25 inches? Ans. 245.4375 sq. in.

4. What is the area of a piece of land in the form of a semicircle, the straight side of which is 40 feet? Ans. 628.32 sq. feet.

5. If a garden, in the form of a half circle, measures 18 feet from the centre of the straight side, to the centre of the curve, what number of square feet does it contain? Ans. 508.9392 sq. feet.

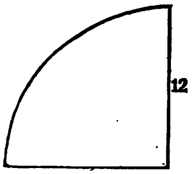
SECTION XVII.

Quadrant of a Circle.

To find the area of a Quadrant of a Circle.

Multiply the square of the radius by .7854.

What is the area of a quadrant of a circle, the radius of which is 12?



12×12=144×.7854=113.0976, the area.

If the arc only be given:

Multiply the square of the arc by .31832, *the product will be the area.*

To find the length of an arc of a Circle.

Multiply the diameter by .7854, the product is the length of a Quadrant.

" radius " " " an arc of 45°

" $\frac{1}{2}$ do. " " " " $22\frac{1}{2}$°

" $\frac{1}{3}$ do. " " " " 15°

" $\frac{1}{5}$ do. " " " " 9°

" $\frac{1}{9}$ do. " " " " 5°

" $\frac{1}{45}$ do. " " " " 1°

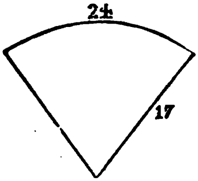
SECTION XVIII.

Sectors of Circles.

To find the area of the Sector of a Circle.

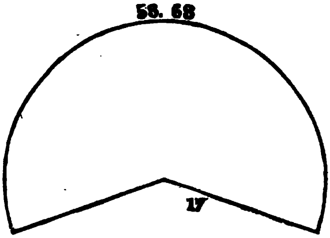
Multiply the arc of the sector by half the radius.

1. What is the area of a sector of a circle, the arc of which is 24 and radius 17?



Half of 17 is 8.5 and 24×8.5=204, the area.

2. What is the area of a sector, the arc of which is 58.68 and radius 17?



Half of 17 is 8.5 and 58.68×8.5=498.78, the area.

3. If the radius of a sector be 12 inches and the arc be 3 feet, what is the area of the sector? Ans. 18 sq. ft.

4. What is the area of a sector of a circle, the diameter of which is 25 and arc 55? Ans. 687.5.

5. What is the area of a piece of land in the form of a sector of a circle, the curved side being 35 rods and the two straight sides being each 15 rods?

Ans. 262½ sq. rods.

6. What is the area of a sector of a circle, the arc of which is 45 degrees and radius 25 inches?

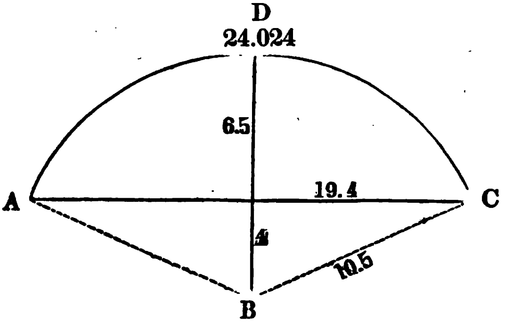
Ans. 245.4375 sq. in.

SECTION XIX.

Segment of a Circle.

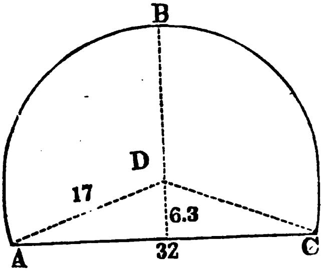
To find the area of a Segment of a Circle.

From the area of a sector of which the segment is a part, subtract the area of the triangle between the chord and radii; the remainder is the area of the segment.



24.024×5.25=126.126, area of the sector A B C D.
19.4×2=38.8, the area of the triangle A B C.
126.126—38.8=87.326, area of the segment A D C.

If the segment is greater than a semicircle, add the area of the triangle to the area of the sector.



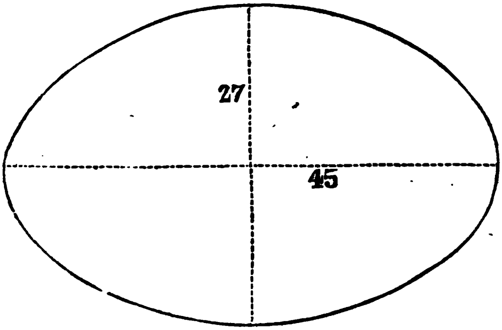
Length of the arc A B C 64.704. Half the radius 8.5.
64.704×8.5=549.984, the area of the sector A B C D.
16×6.3=100.8, area of the triangle A D C.
549.984+100.8=650.784, area of the segment A B C

SECTION XX.

Ellipsis.

To find the area of an Ellipsis.

Multiply the product of the two diameters by .7854.



1. What is the area of an ellipsis, the diameters of which are 43 and 27 inches?

43×27=1161×.7854=911.8494 sq. in.

2. What is the area of an ellipsis, the diameters of which are 25 and 18 inches? Ans. 353.43 sq. in.

3. If the transverse diameter of an ellipsis be 15 feet and the conjugate 10 feet, what will be the area?

Ans. 117.81 sq. feet.

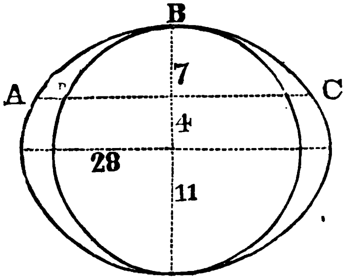
SECTION XXI.

Elliptical Segment.

To find the area of an Elliptical Segment cut parallel to the transverse diameter.

Divide the transverse diameter by the conjugate and multiply the area of a circular segment of the same height by the quotient; the product is the area of the elliptical segment.

1. What is the area of an elliptical segment cut parallel to the transverse diameter, the height of which is 7, and the diameters 28 and 22?



28 divided by 22=1$\frac{3}{11}$; 96.055 area of circular segment D B E.

96.055×1$\frac{3}{11}$=122.252, area of the elliptical segment A B C.

2. What is the area of an elliptical segment cut parallel to the transverse diameter, the height of which is 5 inches, the diameters being 30 and 15 inches?

Ans. 84.1085 sq. in.

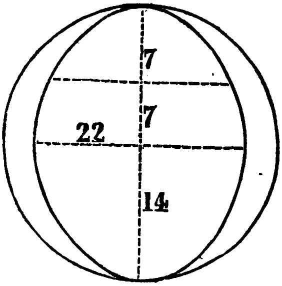
SECTION XXII.

Elliptical Segment.

To find the area of an Elliptical Segment cut parallel to the conjugate diameter.

Divide the transverse by the conjugate diameter, then divide the area of a circular segment of the same height by the quotient, the result will be the area of the elliptical segment.

1. What is the area of an elliptical segment cut parallel to the shorter diameter, the height of which is 7 and diameters 28 and 22?



28 divided by $22 = 1\frac{3}{11}$; 96.055, area of circular segment.

$96.055 \div 1\frac{3}{11} = 75.472$, area of the elliptical segment.

2. What is the area of an elliptical segment cut parallel to the conjugate diameter, the height of which is 7, and diameters 30 and 15 feet? Ans. 21.02717 sq. feet.

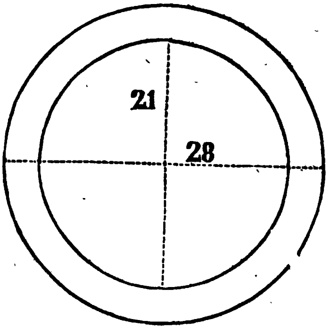
SECTION XXIII.

Circular Annulus.

To find the area of a Circular Annulus, or Ring.

Multiply the difference of the squares of their diameters by .7854, the product will be the area of the circular ring.

1. What is the area of a circular ring, formed by two concentric circles, the diameters of which are 28 and 21?



$28+21\times7=343$, the difference of the squares of the diameters.

$343\times.7854=269.3922$, area of the ring.

2. If the diameters of two concentric circles be 10 and 15 inches, what is the area of the annulus between their circumferences? Ans. 98.175 sq. in.

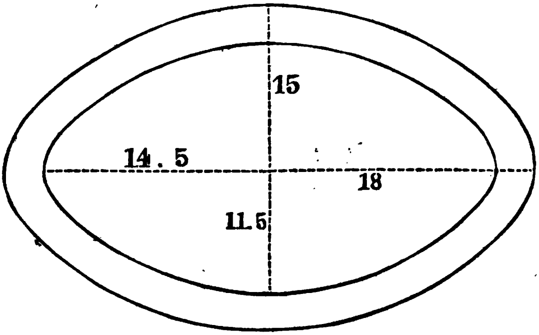
SECTION XXIV.

Elliptical Annulus.

To find the area of an Elliptical Annulus, or Ring.

Multiply the difference of the products of the diameters of the two Ellipses by .7854, the product will be the area of the Elliptical Ring.

What is the area of an elliptical ring formed by two concentric ellipses, the diameters of which are, of the greater 36 and 30, of the smaller 29 and 23?



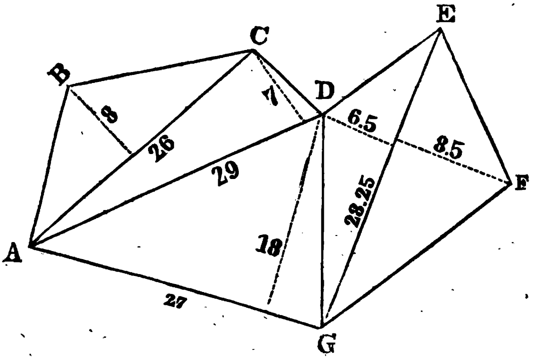
29×23=667.   36×30=1080.
1080—667=413×.7854=324.37, area of the ring.

SECTION XXV.

Irregular Polygons.

To find the area of Irregular Polygons.

Divide the figure into trapeziums and triangles, and the sum of their areas will be the area of the whole figure.



26×4=104. Area of the triangle A B C.
29×3.5=101.5 Area of the triangle A C D.
27×9=243. Area of the triangle A D E.
28.25×7.5=211.875 Area of the trapezium D E F G.

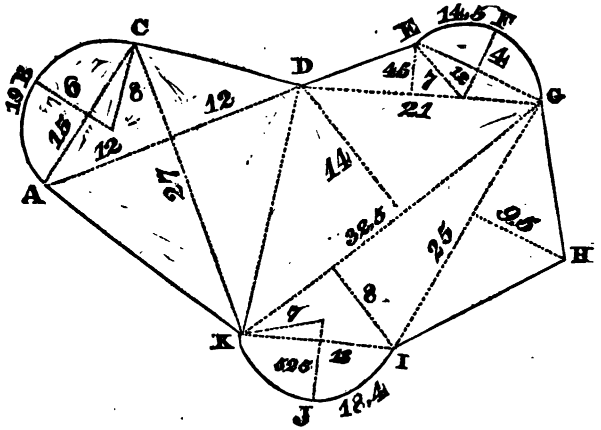
660.375 Area of the polygon A B C D G F E.

SECTION XXVI.

Compound Figures.

To find the area of Compound Figures.

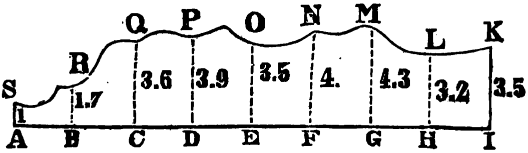
Divide the figure into triangles, trapeziums, and segments of circles, and the sum of their areas will be the area of the whole figure.



61	Area of the segment A B C.
324	Area of the trapezium A C D K.
47.25	Area of the triangle D E G.
32.75	Area of the segment E F G.
357.5	Area of the trapezium D G I K.
118.75	Area of the triangle G H I.
53.025	Area of the segment I J K.
994.275	Area of the whole figure A B C D E F G H I J K.

If one of the boundaries of a compound figure be a right line, the whole extent of the figure, the area may be found by the following rule:

Set off the right line into equal divisions, and erect perpendiculars at the points of division; add all the perpendiculars twice over, except the first and last, which must be taken but once; this sum multiplied by half the equal distance gives the area.



1. =1.
1.7×2=3.4
3.6×2=7.2 25÷2=1.25, half the equal distance.
3.9×2=7.8
3.5×2=7.
4.0×2=8.
4.3×2=8.6
3.2×2=6.4
3.5 =3·5

52.9×1.25=66.125, the area.

SECTION XXVII.

Miscellaneous Examples.

1. What are the areas of an equilateral triangle, a square, and a circle, the perimeter of each of which is 24 inches? Ans. Triangle 27.712 sq. inches.
Square 36. sq. in.
Circle 45.838 sq. in.

2. What is the difference in area between a square and an equilateral triangle, the sides of each of which are 12 inches? Ans. 81.66 sq. in.

3. What is the length of the diameter of a circle, the area of which shall equal that of a square whose side is 15 inches? Ans. 16.92 sq. in.

4. What must be the radius of a circular piece of land which shall equal in area a square, the side of which is 25 feet? Ans. 14.105.

5. What is the length of each side of a square containing one acre? Ans. 208.71.

6. Required the area in square rods of a piece of land in the form of a rhombus, each side of which measures 6.4 chains, and the perpendicular height 5.45 chains?
Ans. 558.08 sq. rods.

7. What will be the cost for painting the four walls of a house that is 40 feet long, 35 feet wide, and 50 feet high, allowing the windows to be whole work, at $12\frac{1}{2}$ cents per square yard? Ans. $104.167.

8. A field in the form of a trapezium measures 1660 feet between the two extreme angles, and the perpendic-

ulars to the remaining angles are 702 and 712 feet; what number of acres does the field contain?

Ans. 26 acres, 7 roods, 14 rods, 133.5 feet.

9. What length of chord, from a central point, fastened to a horse's mouth, will permit him to graze on an acre of grass? Ans. 37.235 feet.

10. Admitting the diameter of the earth to be 7920 miles, what is the circumference? what would be the area of a circle of the same diameter?

Circumference, 24881. miles.
Area of the circle, 49,265,314 sq. miles.

11. How far a part must the points of a pair of dividers be placed, to form a circle containing two square feet? Ans. 9.574 in.

12. What will be the radius of a circular garden containing one quarter of an acre? Ans. 58.875 feet.

13. If 22500 rods of land are to be laid out in the form of a square, what will be the length of one of the sides? Ans. 150 rods.

14. Suppose it be required to draw a circle, the area of which shall be equal to the circumference, what must be the radius? Ans. 2.

15. How many pieces of land, 13 feet square, will be equal in area to one piece, each side of which measures 91 feet? Ans. 49 pieces.

91 divided by 13 is 7×7=49.

16. What is the difference in area between a circle, the radius of which is 15 feet, and a triangle, the base and altitude of which are 15 feet?

Circle 706.86 square feet.
Triangle 112.5 " "

The circle contains 594.36 square feet more than the triangle.

17. If 100 gallons of water be discharged through a pipe 3 inches in diameter in 12 minutes, what must be the diameter of a pipe to discharge 400 gallons with equal velocity in the same time? ANS. 6 in.

18. It is required to lay out a garden on a square piece of land containing 2025 square feet, to be formed with eight equal squares enclosing a circle in the centre; what will be the area of the circle and of each square?

Area of the circle 176.715 sq. feet.
Area of each square 225 sq. feet.

19. A piece of land 40 rods long and 4 rods wide contains one acre, how long must a piece be that is 6¼ rods wide to contain one acre? ANS. 25⅗ rods long.

20. In a house, consisting of four stories, the lower story contains 8 windows of 12 panes of glass, each pane 16 inches long, 12 inches wide; the second and third stories, 8 windows 12 panes each, the glass 15 inches long and 11 inches wide; the forth story, 10 windows 12 panes each, the glass 14 by 10; how many square feet of glass are there in the house? ANS. 464⅔ sq. feet.

21. How many feet of boards will be required for the walls of a house 30 feet 6 inches long, 25 feet 6 inches wide, and 40 feet high, allowing for 20 windows 5½ by 3 feet? ANS. 4150 sq. feet.

22. In a disaster at sea, it became necessary, in order to stop a leak in the ship, to cut out a decayed plank 12 feet long and 3 feet wide; the only sound piece of plank on board measured 9 feet long and four feet wide. To save the vessel, the carpenter contrived to cut the plank into two equal pieces, so as exactly to fill the breach. How was the plank cut?

GEOMETRY.

PART FOURTH.

CHAPTER I.

MENSURATION OF SOLIDS.

SECTION I.

Definitions, &c.

Solids are such bodies as have length, breadth, and thickness.

The mensuration of solid bodies is called cubic measure.

By cubic measure, is determined how many equal square solids or cubes of known dimensions, are contained in any given solid body.

These squares or cubes are one line, one inch, one foot, &c., on each side; called one cubic line, one cubic inch, one cubic foot, &c.

Cubic Measure.

1728	cubic lines	are	1 cubic inch.
1728	cubic inches	"	1 cubic foot.
27	cubic feet	"	1 cubic yard.
231	cubic inches	"	1 gallon, wine measure.
282	cubic inches	"	1 gallon, beer measure.
268.8	cubic inches	"	1 gallon, dry measure.
2150.4	cubic inches	"	1 bushel.

The dimensions of a bushel measure are 18½ inches diameter, and 8 inches deep.

16	cubic feet	are	1 foot of wood.
8	feet of wood	"	1 cord.

The dimensions of a cord of wood are 8 feet long, 4 feet wide, and 4 feet high, or any other form containing 128 cubic feet.

SECTION II.

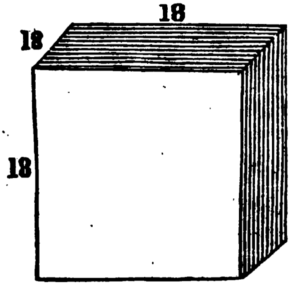
A Cube.

A Cube is a solid having six equal sides, each side being a square plane.

To find the solidity of a Cube.

Multiply the area of one side by its length, and the product will be the solidity of the cube.

1. What is the solidity of a cube, each side of which is 18?



18×18=324, area of one side.
324×18=5832, solidity of the cube.

2. What is the solidity of a cube, each side of which measures 25 inches? Ans. 15625 cubic inches.

3. What is the solidity of a cube, each side of which measures 3 feet, 6 inches? Ans. 42.875 cubic feet.

4. How many 3 inch cubes may be cut from a 12 inch cube? Ans. 64.

5. How much will it cost to dig a cellar 8 feet square and 8 feet deep, at 25 cents per cubic yard?

Ans. \$14.22$\frac{2}{9}$.

6. How many gallons of water will fill a cistern 10 feet square and 10 feet deep?

Ans. 7480$\frac{120}{231}$ gallons, wine measure.
6127$\frac{186}{282}$ gallons, bear measure.

7. How many cubic feet are there in a box which measures 3 feet 6 inches on each side?

The inches may be changed to decimals or duodecimals. Ans. 166$\frac{3}{8}$ cubic inches.

8. How many 4-inch cubes and 8 inch cubes, of each an equal number, may be cut from one 12 inch cube?

Ans. 3 four-inch cubes, and 3 eight-inch cubes.

SECTION III.

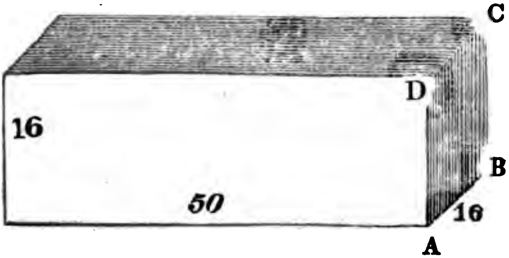
A Parallelopiped.

Parallelopiped is a solid having six sides, the opposite sides being equal and parallel planes.

To find the solidity of a parallelopiped.

Multiply the area of one end by the length, and the product will be the solidity.

1. What is the solidity of parallelopiped, the base of which 16 inches square and length 50 inches?



16×16=256 sq. inches, area of one end A B C D.
256×50=12800 cubic inches, solidity of the parallelopiped.

2. How many cubic feet are there in a timber 20 feet long, the ends of which are 2 feet square?

Ans. 80 cubic feet.

3. In a stick of timber 18 inches square and 15 feet long, how many cubic feet? Ans. 33¾ cub. feet.

If two of the dimensions are given in inches and one in feet, multiply the dimensions together and divide the product by 144; *the quotient will be cubic feet.*

4. What is the solid measure of a mahogany plank 15 feet long, 2 feet wide, and 5 inches thick?

Ans. 12½ cubic feet.

If one of the dimensions be given in inches and the other two in feet, multiply the dimensions together and divide by 12; the quotient will be cubic feet.

15×2=30 sq. feet, this gives the area of the surface.
30×5=150, this gives twelfths of cubic feet.
150÷12=12½ cubic feet.

5. How far from the end must a timber be cut which is 12 inches wide and 8 inches thick, so that the piece may contain 1 cubic foot? Ans. 18 in.

12×8=96. 1728÷96=18

6. How many cubic inches may be placed in a box which measures inside, 4 feet in length, 2½ feet in width, and 3 feet in depth?

4×2½×3=30 cubic feet.
1728×30=51840 cub. in.

1 cubic foot is 1728 cubic inches.

7. How many gallons of water will fill a vessel 4 feet long, 3 feet wide, and 5 feet deep?

Change the cubic feet to cubic inches, and divide by the number of cubic inches in a gallon.

Ans. 448.8 gallons, wine measure.

8. How many cubic feet of earth must be removed, to form a cistern 8 feet square and 10 feet deep?

How many gallons of water will the cistern contain?

Ans. 4787.4 wine gallons.

9. How many cubic feet in a stick of timber 25 feet long, 21 inches wide, and 18 inches thick?

Ans. 65⅝ cubic feet.

SECTION IV.

Wood Measure.

A pile of wood 4 feet long, 4 feet wide, and 4 feet high, is half a cord. Two such piles is a cord.

1. How many cubic feet in a cord of wood?
Ans. 128 cubic feet.

2. How many cords in a pile of wood 12 feet long, 4 feet wide, and 8 feet high? Ans. 3 cords.

3. How many cords in a pile of wood 127 feet long, 16 feet wide, and 20 feet high? Ans. 317½ cords.

4. A pile of wood 4 feet long, 4 feet high, and 1 foot wide, is a foot of wood. How many cubic feet in a foot of wood? Ans. 16 cubic feet.

5. How many feet of wood are there in a cord?
Ans. 8 feet.

6. How many feet of wood are there in a load 8 feet long, 4½ feet wide, and 6 feet high?
Ans. 13½ feet.

Wood for fuel is usually cut 4 feet in length, and placed in carts and wagons in two ranges, one behind the other; the load then is 8 feet long; therefore, in measuring the number of feet of wood, it is unnecessary to multiply by the length when it is exactly 8 feet, for multiplying half of the height by the width gives the number of feet of wood.

From this may be deduced the following rule for measuring loads of wood and bark.

If the load be 8 *feet long, multiply the width by half the height; the product is the number of feet of wood.*

7. How many feet of wood in a load 8 feet long, 5 feet wide, 6 feet high? Ans. 15 feet.

Half of 6 is 3, and 5×3=15.

8 In a load of wood 8 feet in length, 4 feet in width, and 4½ feet in height, how many feet? Ans. 9 feet.

Half the width is 2, 4½×2=9.

9. If a load of wood measures 8 feet long, 5½ feet wide, and 7 feet high, how many feet of wood does it measure? Ans. 19¼ feet.

10. How much will be the cost of a load of pine wood which is 8 feet long, 4½ feet wide, and 9 feet high, at 60 cents per foot? Ans. $12,15.

11. What will be the cost of a load of oak wood which is 8 feet long, 4½ feet wide, and 6 feet high, at 80 cts. per foot? Ans. $10,80.

12. If wood be $6 per cord, how much is it per foot?
Ans. 75 cents.

13. How much will a cord of wood cost at five shillings and three pence per foot? Ans. $7,00.

SECTION V.

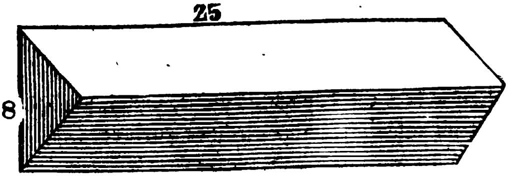
A Prism.

A Prism is a solid, the ends or bases of which are equal and parallel plane triangles or polygons, and the sides plane parallelograms.

To find the solidity of a Prism.

Multiply the area of its base by the length.

1. What is the solidity of a triangular prism, each side of the base being 8 inches and length 25 inches?



Area of the base 27.71×25=692.75 cubic inches.

2. What is the solidity of a triangular prism, each side of the base being 13 inches and length 4 feet?

ANS. 2.033 cub. feet.

3. A pentagonal column measures on each side of the base $9\frac{1}{2}$ inches, perpendicular height 6.5378, and length 6 feet: what is the solidity? ANS. 6.47 cubic feet.

4. What is the solidity of an equilateral triangular column, each side of the base being 54 inches and height 20 feet? ANS. 175.365 cub. feet.

5. An octagonal marble column measures 16 feet periphery and 20 feet altitude: what is the solidity?

ANS. 386.274 cubic feet.

6. How many cubic feet of earth must be removed to form a canal 31 miles long, 20 feet broad at top, 12 feet at the bottom, and 5 feet deep, the ends being perpendicular? ANS. 13094400 cubic feet.

The Erie Canal, in the State of New-York, extends from Albany to Buffalo, 365 miles, in nearly a straight line. It is 40 feet wide at the top and 28 feet at the bottom: how many cubic feet of water will it contain — how many hogsheads, of 120 gallons each?

SECTION VI.

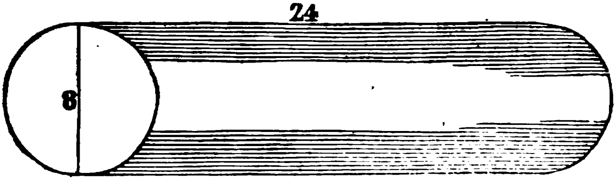
A Cylinder.

A Cylinder is a solid, with circular, equal, and parallel bases, and its sides a regular circular surface.

To find the solidity of a Cylinder.

Multiply the area of one of the bases by the length, the product will be the solidity.

1. What is the solidity of a cylinder, the diameter of the base being 8 inches and length 24 inches?



8×8×.78539=50.265, area of the base.

50.265×24=1206.36 cubic inches, the solidity of the cylinder.

2. What is the solidity of a cylinder 10 feet in length and 2 feet diameter? Ans. 31.416 cubic feet.

3. What is the solidity of a circular stick of timber 12 feet long and 15 inches diameter?

Ans. 14.72 cubic feet.

4. How many cubic feet in a circular marble column 20 feet in length, the diameter of each base 2.5 feet?

Ans. 98.175 cubic feet.

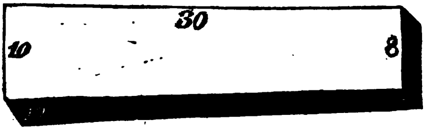
5. How many gallons of water will fill a circular vessel 2 feet diameter and 4 feet deep? Ans. 94 gallons.

6. How many gallons of water will a reservoir contain which is 16½ feet diameter and 13 feet deep?

Ans. 20793 gallons.

To find the solidity of a body, the bases or ends of which are of unequal breadth for thickness.

Half the sum of the areas of the two bases or ends is the mean area: multiply the mean area by the length, the product is the solidity.



1. What is the solidity of a body 30 inches in length, the greater base 10 inches, and the less 8 inches square?

Ans. 2460 cubic inches.

10×10=100 and 8×8=64; 100+64=164; half of 164 is 82; 82×30=2460.

2. What is the solidity of a piece of marble 2 feet in length, the greater base 5 inches square, and less base 5 inches wide and 3 inches thick? Ans. 480 cub. in.

5×5=25 and 5×3=15; 25+15=40; half of 40 is 20; 20×24=480.

SECTION VII.

A Pyramid.

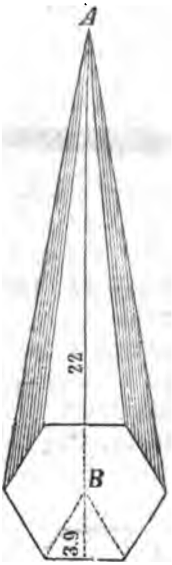
A Pyramid is a solid, the base of which is a regular polygon, and its sides plain triangles meeting at their vertices.

To find the solidity of a Pyramid.

Multiply one third of the area of the base by the altitude, the product is the solidity.

This changes the pyramid to a prism of the same altitude.

1. What is the solidity of a hexagonal pyramid, the base of which is 4.5 on each side, perpendicular 3.9, and altitude 22?



The altitude of a pyramid is the perpendicular distance from the centre of the base to the apex from B to A.

Every pyramid is equal to one third of a prism, having an equal base and altitude.

Area of the base 526.5, one third of this is 175.5×22 =3861 cubic feet.

2. Each side of the base of a triangular pyramid is 4 feet and altitude 15 feet: what is the solidity?

Ans. 34.64 cubic feet.

3. What is the solidity of a square pyramid, each side of the base being 36 inches and altitude 10 feet?

Ans. 30 cubic feet.

4. What is the solidity of a marble pyramid, the base of which is a pentagon, each side of which is 48 inches and perpendicular 33.0332 inches, and altitude 15 feet?

Ans. 27.6276 cubic feet.

5. A hexagonal pyramid is 5 feet on each of the base, and altitude 25 feet: what is the solidity?

Ans. 13 cubic feet.

6. One of the largest pyramids in Egypt has a square base 26 feet 4 inches on each side, and 499 feet high: how many cubic feet does it contain?

Ans. 115.343 cubic feet.

SECTION VIII.

A Cone.

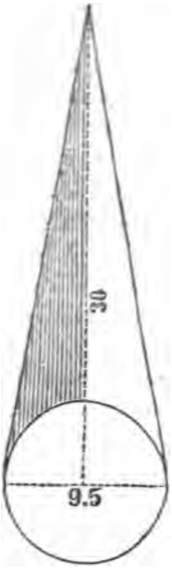
A Cone is a pyramid, the base of which is a circle, and the sides a uniformly decreasing circular surface.

To find the solidity of a cone.

Multiply one third of the area of the base by the altitude.

This changes the cone to a cylinder of the same altitude.

1. What is the solidity of a cone, the diameter of the base being 9.5 and altitude 30?



9.5×9.5×.7854=70.88235, $\frac{1}{3}$ of this is 236.2745×30=708.8235, the solidity.

The solidity of a cone is one third of the solidity of a cylinder of equal base and altitude.

2. The diameter of the base of a cone is 12 inches and the altitude 48 inches: what is the solidity?

Ans. 1.0472 cubic feet.

3. What is the solidity of a marble cone, the diameter of the base of which is 18 inches and altitude 4 feet?

Ans. 6.5156 cubic feet.

4. How many cubic feet are there in a stone monument in the the form of a cone, 6 feet diameter and 20 feet high? Ans. 18.849 cubic feet.

SECTION IX.

Frustum of a Pyramid.

The frustum of a pyramid is the part remaining after the top has been cut off parallel to the base.

To find the solidity of a Frustum of a Pyramid.

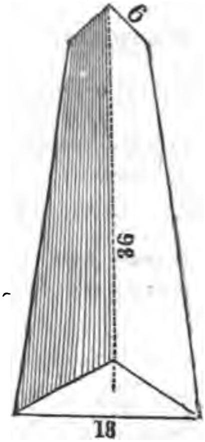
General Rule.

Multiply the mean area by the altitude; the product is the solidity.

If the bases are equilateral triangles,

To the product of one side of the greater base multiplied by one side of the less base, add one third of the square of their difference, this sum multiplied by .433013 *gives the mean area, which, multiplied by the altitude gives the solidity.*

1. What is the solidity of a frustum of a pyramid, the bases of which are equilateral triangles, the sides of the greater base 18, side of the less base 6, and altitude of the pyramid 36?



18×6=108, product of one side of each of the two bases.

18—6=12×12=144, $\frac{1}{3}$ of this is 48. 108+48= 156×.433013=67.550028, the mean area, which, multiplied by 36, gives 2431.8 for the solidity of the frustum.

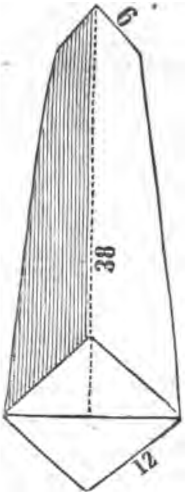
2. The frustum of a triangular pyramid, on each side of the greater base is 24 inches, each side of the less base 12 inches, and altitude 8 feet; what is the solidity? Ans. 8.0829 cubic feet.

3. What is the solidity of a triangular column 25 feet high, the bases equilateral triangles, the greater 3 feet and less 1$\frac{1}{2}$ feet on each side? Ans. 56.83 cubic feet.

To find the solidity of a frustum of a pyramid having square bases.

To the product of one side of each of the two bases, add one third of the square of their difference; this gives the mean area, which multiplied by the altitude gives the solidity.

1. What is the solidity of a frustum of a square pyramid, the sides being 12 and 6, and altitude 38?



12×6=72; 12—6×6×6=36; one third of 36 is 12; 72 and 12 are 84, this multiplied by the height, 84×38 =3192, the solidity.

2. What is the solidity of a stick of timber 25 feet in length, the ends being 10 inches and 13 inches square?

Ans. $23\frac{13}{144}$ cubic ft.

3. What is the solidity of a marble pillar 10 feet in height, the bases squares, each side of the greater base 2 feet, and of the less base 18 inches?

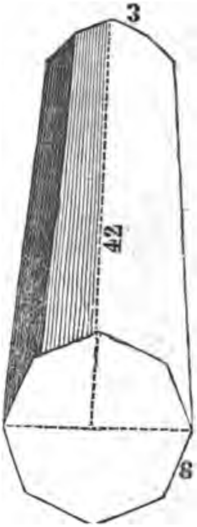
ANS. $30\frac{5}{6}$ cubic feet.

4. What is the solidity of a granite column 50 feet altitude, and bases squares, the sides being 7 feet and 4 feet? ANS. 1550 cub. feet.

To find the solidity of a frustum of a pyramid, the bases of which are regular octagons.

To the product of a side of each of the two bases, add one third of the square of their difference. This sum multiplied by 4.828427 *gives the mean area, which multiplied by the height gives the solidity.*

1. What is the solidity of a frustum of a pyramid having octagonal bases, each side of the greater base being 8, each side of the less base 3, and altitude 42?



8×3=24, and 8—3=5×5=25; $\frac{1}{3}$ of 25 is 8$\frac{1}{3}$; 24 and 8$\frac{1}{3}$=32$\frac{1}{3}$; 32$\frac{1}{3}$×4.828427 gives 156.12014; this multiplied by 42 gives 6557.04588 for the solidity.

2. What number of cubic feet are there in a marble column with octagonal bases, the perimeters of the bases being 120 inches and 80 inches, and altitude 5 feet?

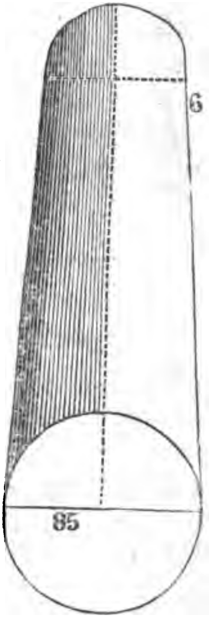
Ans. 26.58024 cubic feet.

3. What is the solidity of a stone pillar 30 feet altitude, with hexagonalbases, the perimeters of which are 12 feet and 30 feet? Ans. 1883.086 cubic ft.

To find the solidity of a frustum of a cone.

To the product of the two diameters, add one third of the square of their difference; this sum multipled by .7854 gives the mean area. Multiplying the mean area by the altitude will give the solidity.

1. What is the solidity of a frustum of a cone, the diameters of the two bases being 22 and 16, and altitude 38?



22×16=352, and 22—16=6; the square of 6 is 36; ⅓ of 36 is 12; 352 and 12 are 364×.7854=285.8856, the mean area, which multiplied by 38 gives 10863.6528, the solidity.

2. What is the solidity of the frustum of a cone, the diameters of the bases being 21 and 13 inches, and altitude 5 feet? Ans. 8.0267 cubic feet.

3. How many gallons of water will fill a tub 18 inches diameter at bottom, 12 inches diameter at top, and 36 inches deep? Ans. 27.9 gallons.

4. How many cubic feet of water will a circular cistern contain which is 8 feet diameter at the bottom, 10 feet diameter at top, and 12 feet deep?

5. A common pail is 11 inches diameter at the top,

Ans. 766.55 cubic ft.

10 inches diameter at the bottom, and 8 inches deep; how many gallons of water will fill it? Ans. 3 gallons.

SECTION X.

A Sphere.

A Sphere is a solid body, every part of the surface of which is equally distant from the centre.

To find the solidity of a sphere.

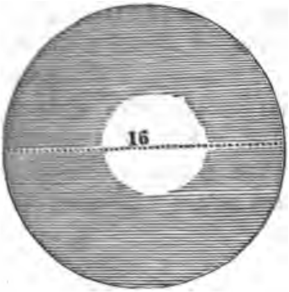
Rule 1.

Multiply the square of the diameter by 1.0472; *this product multiplied by the radius gives the solidity.*

Rule 2.

Multiply the cube of the diameter by .5236, *the product will be the solidity.*

1. What is the solidity of a sphere, the diameter of which is 16 inches?



By Rule 1.

16×16=256×1.0472×8=2144,665 cubic inches.

By Rule 2.

16^3=4096×.5236=2144.665 cubic inches.

2. What is the solidity of a sphere 12 inches in diameter? Ans. 904.78 cubic in.

3. If the diameter of a sphere be 1 inch, what is the solidity? Ans. .5236 of a cubic in.

4. What is the solidity of a silver sphere 5 inches in diameter? Ans. 65.45 cubic in.

5. What is the solidity of a sphere 15 inches in diameter? Ans. 1767.15 cubic in.

6. What is the number of cubic miles in the moon, supposing the diameter to be 2160 miles?

Ans. 5,276,684,625 cubic miles.

7. Supposing the earth to be a sphere, and the diameter to be 7920 miles, what is the solidity in cubic miles?

Ans. 260121860876 cubic miles.

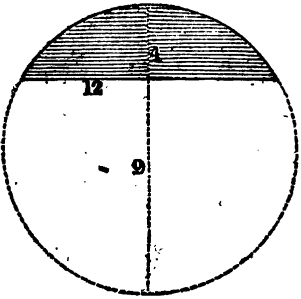
SECTION XI.

Segment of a Sphere.

To find the solidity of a Segment of a Sphere.

To three fourths of the square of the base, add the square of the height; this sum multiplied by the height, and the product by .5236 *will give the solidity of the segment.*

1. What is the solidity of a segment of a sphere, the diameter of which is 12, and height 4?



$12 \times 12 = 144 \times \frac{3}{4} = 108$; $4 \times 4 = 16$; $108 + 16 = 124$; $124 \times 4 = 496 \times .5236 = 259.7$, the solidity.

2. What is the solidity of a spherical segment, the height of which is 3 inches and base 5 inches?

Ans. 43.589 cubic in.

3. If the base of a segment of a sphere be two feet, and height 18 inches, what is the solidity?

Ans. 4.12345 cubic feet.

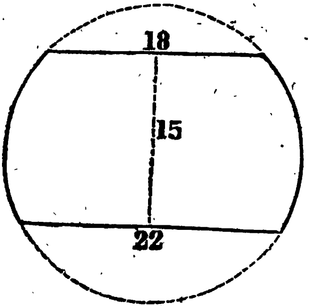
SECTION XII.

Frustum of a Sphere.

To find the solidity of a frustum of a sphere.

To the squares of the radii of the two bases, add one third the square of the height; this sum multiplied by the height, and the product by 1.5708 gives the solidity.

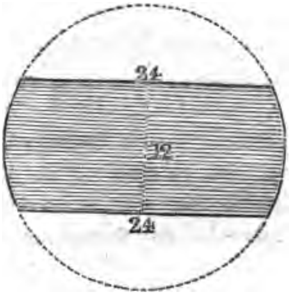
1. What is the solidity of a frustum of a sphere, the diameters of which are 18 and 22 inches, and perpendicular distance 15 inches?



$9\times9=81$
$11\times11=121$
$15\times15\div3=75$

$277\times15=4155\times1.5708=$
6526.674 cub. in

2. What is the solidity of a zone or middle frustum of a sphere, the diameters of the bases being 24, and altitude 12 inches?



$12 \times 12 = 144$
$12 \times 12 = 144$
$12 \times 12 \div 3 = 48$

$336 \times 12 = 4032 \times 1.5708 =$
668.58 cubic inches.

3. What is the solidity of the middle frustum of a sphere, the greater diameter 12 inches, less diameter 10 inches, and height 4 inches? ANS. 416.785 cubic in.

4. If the diameters of the bases of a zone are 4 feet and 5 feet, and height 3 feet, what is the solidity?
ANS. 62.1565 cubic feet.

SECTION XIII.

A Spheroid.

A Spheroid is an oblong solid, formed by the revolution of an ellipsis about its transverse or conjugate diameter.

If the revolution be on its transverse diameter, the solid is called a *prolate spheroid;* if the revolution be on its conjugate diameter, the solid is an *oblate spheroid.*

In a prolate spheroid, the transverse diameter is the axis.

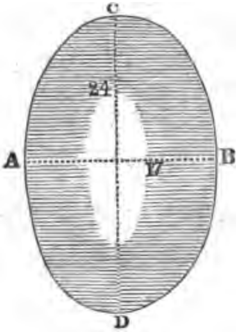
In an oblate spheroid, the conjugate diameter is the axis.

To find the solidity of a Spheroid:

Multiply the square of the revolving diameter by its axis, and the product by .5236 for the solidity.

1. What is the solidity of a prolate spheroid, the diameters of which are 17 and 24?

A Prolate Spheroid.

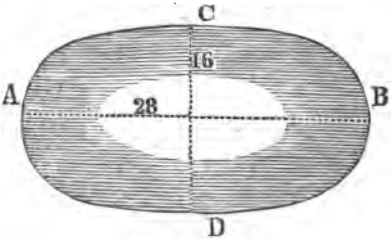


A B revolving diameter, C D axis.

$17\times17=289\times24=6936\times.5236=3631.689$ solidity.

2. What is the solidity of an oblate spheroid, the diameters of which are 28 and 16?

An Oblate Spheroid.



$28\times28=784\times16=12544\times.5236=6568.03$, the solidity.

A B revolving diameter, C D axis.

3. What is the solidity of an oblate spheroid, the diameters of which are 8 and 12 inches?

Ans. 6031.87 cubic in.

4. If the axis of a prolate spheroid be 18 inches, and the revolving diameter 12 inches, what is the solidity?

Ans. 1357.17 cubic in.

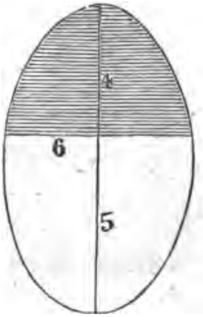
SECTION XIV.

Segment of a Spheroid.

To find the solidity of the Segment of a Spheroid.

From the product of three times the axis, subtract twice the height of the segment; multiply the remainder by the square of the height, and the product by .5236. Then, as the square of the axis is to the square of the revolving diameter, so is the last product to the solidity.

1. What is the solidity of a segment of a prolate spheroid, the axis of which is 9, and revolving diameter 6, and height 4?



9×3=27; 4×2=8; 27−8=19×16=304×.5236 =159.1744; 9×9=81 and 6×6=36.

As 81 is to 36 so is 159.1744 to 70.7447, the solidity.

SECTION XV.

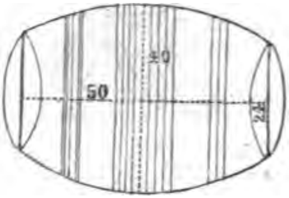
Middle Frustum of a Spheroid.

To find the solidity of the Middle Frustum of a Spheroid.

To twice the square of the middle diameter, add the square of the diameter of the base; this sum multiplied by the perpendicular distance between the bases, and the product by .2618 *gives the solidity.*

This rule may be useful in gauging. A cask of this form is called a cask of the first variety.

1. What is the solidity of the middle frustum of a spheroid, the middle diameter 40, the ends 24, and length 50 inches?



40×40=1600×2=3200; 24×24=576; 3200+576 =3776×50=188800×.2617=40931.84 cubic inches. This divided by 231 will give gallons=177.19 gallons.

SECTION XVI.

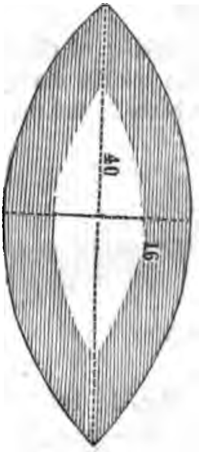
Parabolic Spindle.

A Parabolic Spindle is an oblong solid, the extremities of which are vertices.

To find the solidity of a Parabolic Spindle.

Multiply the square of the conjugate diameter by the transverse, and the product by .41888 for the solidity.

1. What is the solidity of a parabolic spindle, the two diameters of which are 40 and 16?



16×16=256×40=10240×41888=428.933, the solidity.

SECTION XVII.

Middle Frustum of a Parabolic Spindle.

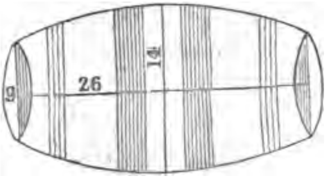
To find the solidity of the Middle Frustum of a Parabolic Spindle.

To twice the square of the middle diameter, add the square of the diameter of the base; from this sum subtract four tenths of the square of the difference of the two diame-

ters; multiply the remainder by the length, and the product by .2168 for the solidity.

A frustum of a parabolic spindle is the form of a cask of the second variety.

1. What is the solidity of the middle frustum of a parabolic spindle, the middle diameter of which is 14, the diameters of the bases 9, and the length 26?



14×14=196; 9×9=81; 196+81=277; difference of the two diameters 14—9=5×5=25; $\frac{4}{10}$ of 25 is 10; 277—10=267; 267×26=6942×.2168=1505.0256, the solidity.

SECTION XVIII.

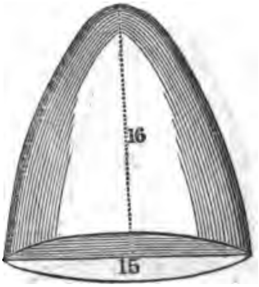
Parabolic Conoid.

A Parabolic Conoid is a solid formed by the revolution of a parabola about its abscissa.

To find the solidity of a Parabolic Conoid:

Multiply the height by the square of the diameter of the base, the product multiplied by .3927 gives the solidity.

1. What is the solidity of a parabolic conoid, the height being 16 inches, and diameter of the base 15?



15×15=225×16=3600×.3927=1413.72.

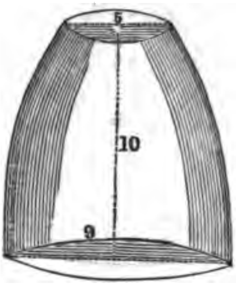
SECTION XIX.

Frustum of a Parabolic Conoid.

To find the solidity of a Frustum of a Parabolic Conoid.

Multiply the sum of the squares of the two bases by the height, and the product by .3927 gives the solidity.

1. What is the solidity of a frustum of a parabolic conoid, the diameter of the two bases being 9 and 5, and height 10?



5×5=25, and 9×9=81; 81 and 25 are 106×10= 1060×.3927=41.626 solidity.

If two frustums of a parabolic conoid of equal bases be united at their greater bases, the form will be a *cask* of the third variety.

SECTION XX.

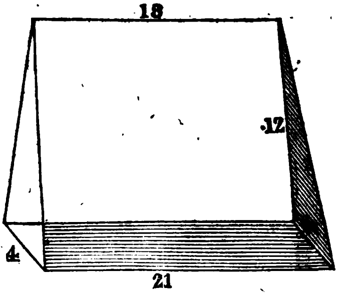
The Wedge.

A Wedge is a solid, the base of which is a square or right angle parallelogram, with two of its sides meeting at an edge.

To find the solidity of a wedge:

To one third of the length of the base add one sixth of the length of the edge; this sum multiplied by the height and thickness gives the solidity.

1. What is the solidity of a wedge, the base of which is 18 inches by 4 inches, the edge 18 inches, and altitude 10 inches?



One third of the base 21 is 7; one sixth of the edge 18 is 3; $7+3=10\times12\times4=480$, the solidity.

2. What is the solidity of a wedge, the base being 24 inches long, 5 inches wide, the edge 18 inches, and the height 15 inches? Ans. 825 cub. in.

3. What is the solidity of a wedge 3 feet long, 12 inches wide at the edge, and base 15 inches by 6 inches? Ans. 1512 cubic inches.

SECTION XXI.

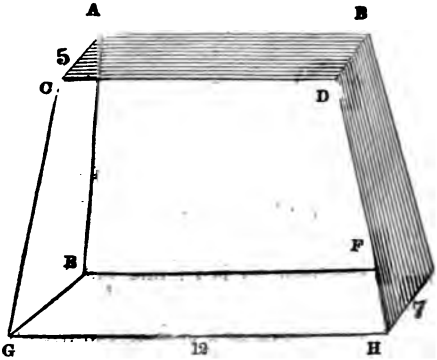
A Prismoid.

A Prismoid is a solid resembling a prism; its bases are rectangles, and its sides four plain trapezoids.

To find the solidity of a prismoid.

If a Prismoid be divided diagonally between the bases, *it will form two wedges. It may, therefore, be measured as such, and the sum of the two wedges will be the solidity of the prismoid.*

1. What is the solidity of a prismoid, the bases of which are 9 by 5 and 12 by 7, and the height 15?



$\frac{1}{3}$ of 9 is 3; $\frac{1}{6}$ of 12 is 2; 3 and 2 are 5; $5\times15\times5=375$, solidity of the wedge A B C D E F; then $\frac{1}{3}$ of 12 is 4; $\frac{1}{6}$ of 9 1.5; 4 and 1.5 are 5.5; $5.5\times15\times7=577.5$, the solidity of the wedge A D E F G H; adding the solidities of the two wedges 375 and $577.5=952.5$, the solidity of the prismoid A B C D E F G H.

2. How many cubic feet in a block of marble, the lower base of which is 42 by 15 inches, and upper base 36 by 12 inches, and altitude 18 inches?

Ans. 9504 cub. in.

SECTION XXII.

Regular Bodies.

A Regular Body is a solid, the surface of which is formed of similar and equal planes.

There are six regular bodies, the names of which are derived from the number of planes which form their surfaces, viz:

1. The Tetraedron has 4 equal and equilateral triangular faces.
2. The Hexaedron or cube, has six equal square faces.
3. The Octaedron has 8 equal and equilateral triangular faces.
4. The Dodecaedron has 12 equal and equilateral pentagonal faces.
5. The Icosaedron has 20 equal and equilateral triangular faces.

The Sphere is the only other regular solid.

Table of the solidity and superficies of the five regular bodies, the length of a side of each being one.

Bodies.	*Solidity.*	*Superficies.*
Tetraedron.	.11785	1.73205
Hexaedron.	1.	6.
Octaedron.	.4714	3.4641
Dodecaedron.	7.663119	20.6457
Icosaedron.	2.181695	8.66025

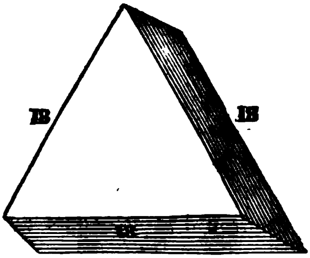
General Rule.

To find the solidity of each of the five regular bodies:

Multiply the cube of a side by the tabular solidity, the product will be the solidity of the body.

Tetraedron.

1. What is the solidity of a tetraedron, each side of which is 18?

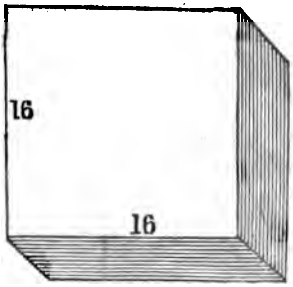


$18^3=5832\times.11785=687.3$, the solidity.

2. If the side of a tetraedron be 5 inches, what is its solidity? Ans. 14.731 cubic in.

Hexaedron.

1. What is the solidity of a hexaedron, each side of which is 16?

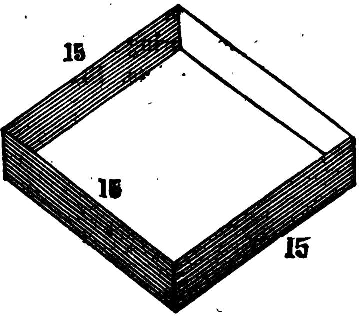


The hexaedron is a cube, and is measured as a cube.

The cube of $16^3=4096$, the solidity.

Octaedron.

1. What is the solidity of an octaedron, each side of which is 15?

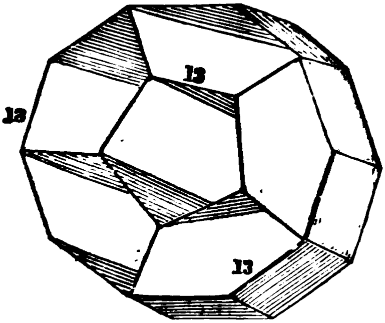


The cube of 15 is $3375\times.4714=1590.975$, the solidity.

2. How many cubic inches are there in a block of marble, in the form of an octaedron, the length of each side being 12 inches? Ans. 814.579 cub. in.

Dodecaedron.

1. What is the solidity of a dodecaedron, each side of which is 13?



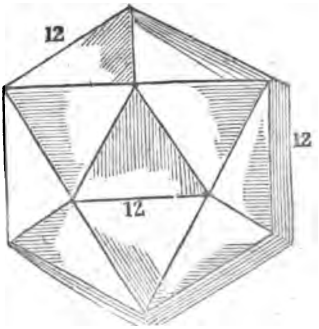
The cube of 13 is 2197×7.663119 gives 16835.872, solidity.

2. What is the solidity of a block in the form of a dodecaedron, each side measuring 12 inches?

Ans. 13141.969 cub. in.

Icosaedron.

1. What is the solidity of an icosaédron, each side being 12?



The cube of 12 is 1728, which multiplied by 2.181695 gives 3769.97, solidity.

2. What is the solidity of an icosaedron, each side being 10 inches? Ans. 2181.695 solidity.

SECTION XXIII.

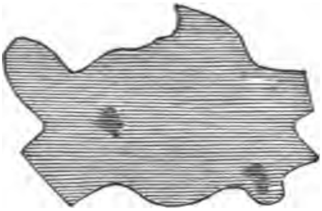
Irregular Bodies.

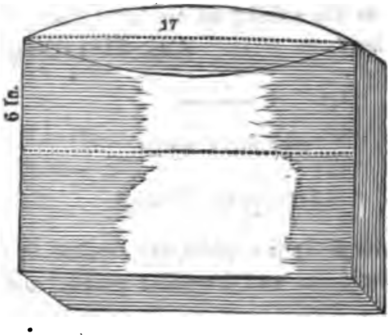
An Irregular Body is a solid, the surface of which is formed of dissimilar and irregular plane, and curved faces.

To find the solidity of Irregular Bodies.

Place the body in any vessel of regular form, and fill the vessel with water or fine sand; then remove the body, and measure the number of cubic inches or feet in the *space which the water or sand has fallen, and it will equal the solidity of the body.*

1. The following figure represents an irregular solid, which, when removed from a circular vessel 17 inches in diameter, in which it had been submerged in water, the fluid fell 6 inches. What was the solidity of the body?





17×17×289×.7854=226.98×6=1361.88 cubic inches, the solidity of the body.

2. A body was placed in a vessel 18 inches long and 12 inches wide, and covered with water; when taken out the water fell 8 inches, what was the solidity?

Ans. 1 cub. foot.

3. Eight apples and four pints of sand were put in a wine gallon measure and filled it; what was the solidity of the apples? Ans. $115\frac{1}{2}$ cub. in.

This is a very convenient method of measuring the solidities of small bodies, and parts of bodies.

4. A small, but irregular piece of mineral was suspended in a tumbler of water, 4 inches in diameter, and raised the fluid $1\frac{1}{2}$ inch; what was its solidity?

Ans. 18.849 cubic in.

5. What is the solidity of a body, which, being placed in a circular vessel 20 inches diameter, caused the water to rise 5 inches? Ans. 1570.8 cubic in.

CHAPTER II.

SECTION I.

To find the area of the surface of a Cube.

Multiply the square of one side by 6, the product will be the area of the cube.

1. What is the area of the surface of a cube, the side of which is 12 inches?

The square of 12 is 144, which, multiplied by 6, the number of sides, gives 864 square inches for the area.

2. If the side of a cube be 3 feet 6 inches, what is the area of the surface? Ans. 73.5 sq. ft.

3. What will it cost to gild the surface of a cube, each side of which measures 25 inches, at $1,25 per. square foot? Ans. $30.05.

SECTION II.

To find the area of the the surface of a Parallelopiped.

Multiply the sum of the sides of the base by the length, and to the product add the areas of the two bases.

1. What is the area of the surface of a parallelopiped, the length of which is 8 inches, breadth 5 inches, and thickness 3 inches?

$5+5+3+3=16$, sum of the sides of the base; $16\times 8=128$; $5\times 3\times 2=30$, areas of the two bases; $128+30=158$, area of the parallelopiped.

2. How many square feet of surface are there on a body 12 feet long, 5 feet broad, and 4 feet thick?

Ans. 256 square feet.

3. What is the area of a parallelopiped 3 feet in length, and the bases 10 inches square?

Ans. 1640 square inches.

SECTION III.

To find the area of the surface of a Prism.

Multiply the circumference of the base by the length, and to the product add the areas of the bases.

1. What is the area of the surface of a triangular prism, each side of the base being 4 inches, and length 15 inches?

Circumference of the base 12 inches; 12×15=180. Areas of the bases 13.856; 180+13.856=193.856 sq. feet.

2. How many square feet are there on the surface of a body 12 feet in length, each side of the base being 2 feet?

The parallelopiped is a square prism, and may be measured as such. Ans. 104 sq. ft.

3. What is the area or superficial measure of a pentagonal marble column 18 feet in length, each side of the base of which is 18 inches? Ans. 142.742 sq. ft.

4. How much will it cost to paint the entire surface of a hexagonal pillar 25 feet in length, each side of the base being 2½ feet, at 12½ cts. per square yard?

Ans. $5,66.

5. What is the area of the surface of a heptagonal solid 20 inches in length, each side of the base 3 inches?

Ans. 485.41 sq. in.

6. How much will be the expense of painting 20 octagonal columns, 17 feet long, 15 inches each side of the base, at 15 cts. per square yard, including only one of the bases of each column? Ans. $59,18.

7. How many square feet on the surface of a marble prism in the form of a decagon, 5 inches each side of the base, and 3 feet long? Ans. 15.17 sq. ft.

8. What is the area of the surface of a dodecagon, each side of the base 2 inches, and length 2 feet?

Ans. 665.69 sq. in.

9. How many square inches on the surface of a cylinder 15 inches in length, and diameter of the bases 5 inches?

The cylinder is a circular prism, and is measured in the same manner. ANS. 274.89 sq. in.

The area of the surface of a cylinder, whose altitude and diameter are equal, is equal to six times the area of a circle of the same diameter.

Six times the square of the diameter, multiplied by .7854, is the area of a cylinder, of which the altitude and diameter are equal.

SECTION IV.

To find the area of the surface of a Pyramid.

Multiply the circumference of the base by half the length of a side, and to the product add the area of the base.

NOTE. The length of a side of a pyramid is the distance from the centre of a side of the base to the vertex.

1. What is the area of the surface of a square pyramid, each side of the base being 10 inches, and the length of each side of the pyramid 36 inches?

10×4=40, circumference of the base; ½ of 36 is 18, half the length of a side; 40×18=720, area of the upright surface; 10×10=100, area of the base; 720+100=820 square inches, the area of the whole pyramid.

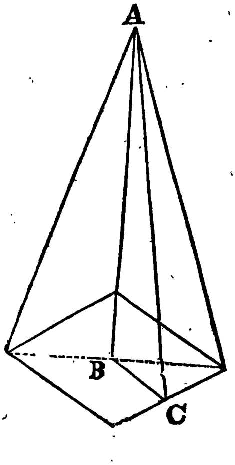
2. In a triangular pyramid, each side of the base of which is 8 inches, and the length of a side of the pyramid 2 feet; what is the area of the surface?

ANS. Upright surface, 288.; area of the base, 27.71: Surface of the entire pyramid, 315.71 sq. in.

It is often necessary to calculate the length of a side of a pyramid.

The altitude of a pyramid may be considered as one side of a right triangle; and the distance from the centre of the base to the centre of a side of the base, as the other side of a right triangle, and the length of a side of a pyramid as the hypothenuse. Therefore, to the square of the altitude add the square of the distance from the centre of the base to the centre of a side of the base, and the square root of the sum will be the length of a side of the pyramid.

3. What is the length of a side of a pyramid having a square base, each side of which is 8 inches, and altitude of the pyramid 20 inches?



The line A B is the altitude, 20; B C the distance from the centre of the base to the centre of the side, 4; A C is the length of a side of the pyramid.

The square of the altitude A B, is 400; the square of B C is 16; 400+16=416; the square root of which is 20.7364, the line A C.

4. What is the length of a side of a triangular pyramid, the altitude of which is 15 feet, and each side of the base 5.8 feet? Ans. 15.175 feet.

5. What is the superficial measure of the triangular pyramid, the dimensions of which are given in the last example? Ans. 147.28 sq. feet.

6. What is the area of the surface of an octagonal pyramid, the perimeter of the base of which is 16 feet, and altitude 6 feet? Ans. 67.313 sq. feet.

7. What is the area of the entire surface of a circular pyramid, or cone, the diameter of the base being 12 inches, and altitude 3 feet? Ans. 2.2854 square feet.

The surface of any pyramid is equal to one half the surface of a prism of the same base and altitude.

SECTION V.

To find the area of the surface of a frustum of a pyramid.

Multiply the sum of half the perimeters of the two bases by the length of a side, and to the product add the areas of the two bases.

1. What is the rea of the surface of a frustum of a square pyramid, the side of the greater base, 12 feet; of the less base, 9 feet; and the length of a side of the frustrum, 18 feet?

Area of the sides, 189; area of the greater base, 144; area of the less base, 81. The sum of the whole is 414 square feet.

2. If the length of a side of a triangular pyramid be 10 feet, and each side of the greater base 1 foot, and of the less 9 inches, what is the area of the surface?

Ans. 9.426585 sq. feet.

3. What is the superficial measure of the frustum of an octagonal pyramid, the periphery of the bases being 16 feet and 12 feet, and length of the sides 30 feet?

Ans. 31.3847 sq. feet.

4. What is the area of the surface of a frustum of a cone, or circular pyramid, the altitude of which is 25 feet, and the diameters of the bases 3 feet and 5 feet?

Ans. 126.7 sq. feet.

SECTION VI.

To find the area of the surface of a sphere or globe.

The area of a sphere is four times the area of a circle of equal diameter: therefore, multiply four times the square of the diameter by .7854; *the product will be the area of the sphere; or, Multiply the square of the diameter by* 3.14159, *the product will be the area of a sphere.*

1. What is the area of a sphere, the diameter of which is 5 inches?

Square of the diameter, 25;

$25 \times 4 = 100 \times .7854 = 78.540$ Ans. 78.54 sq. in.

2. What is the area of a sphere, the diameter of which is 15 feet? Ans. 706.86 sq. feet.

The areas of spheres are in proportion to each other, as the squares of their radii, or diameters.

3. If the surface of a sphere, 5 inches in diameter, be

78.54, how many times as large must be the surface of a sphere, 10 inches in diameter?

The square of 5 is 25; the square of 10 is 100; 100 is 4 times 25.

ANS. 4 times as large = 314.16 square inches.

4. If it cost 5 dollars to gild a sphere 4 inches in diameter, how much will it cost to gild one which is 8 inches in diameter? ANS. 20 dollars.

5. What is the area of a sphere, the radius of which is 3 feet 6 inches?

3 ft. 6 in. radius, the diameter is 7 ft.; the square of 7 is 49; 3.1416×49=155.9384 sq. inches, the surface.

6. If the area of a circle be 15 inches, what will be the area of a sphere of equal diameter? ANS. 60 in.

The area of the entire surface of a hemisphere is three-fourths of the area of a sphere of the same diameter.

7. If it cost 24 dollars to gild the surface of a sphere, how much will it cost to gild the surface of a hemisphere of the same diameter? ANS. 18 dollars.

8. What is the area of a hemisphere, 10 inches in diameter? ANS. 235.62 square inches.

9. What is the difference in area between the surface of a sphere and the surface of a hemisphere, each 25 inches in diameter?

Area of the sphere,	1963.49
Area of the hemisphere,	1472.62
Difference in area,	490.87 sq. inch.

If the circumference of a sphere be given, to find the area of the surface.

Multiply the square of the circumference by .31832, *the product will be the area of the surface.*

1. What is the surface of a sphere, the circumference of which is 5 inches?

$5\times5=25\times.31832=7958$ sq. inches.

SECTION VII.

To find the area of each of the five regular solids.

Multiply the area of one side by the number of sides in the solid; the product will give the area of the whole.

1. What is the area of the surface of a tetraedron, each side of one of the planes being 12 inches?

If each side of an equilateral triangle be 12, the area will be 62.354; this multiplied by 4, the number of sides, gives 249.416 sq. inches, the area.

2. What is the surface of a hexaedron, each side of which is 10 inches? ANS. 600 sq. inches.

3. What is the area of an octaedron, each side being 10 inches? ANS. 346.41 sq. inches.

4. What is the area of a dodecaedron, each side of one of its pentagonal faces being 10 inches?

ANS. 2064.57 square inches.

5. What is the area of an icosaedron, each side of one of its triangular faces being 10 inches?

ANS. 866. sq. in.

CHAPTER III.

MISCELLANEOUS QUESTIONS.

SECTION I.

1. How many square inches of leather will cover a ball 5 inches in diameter? Ans. 78.54 sq. in.

2. If it be required to make a sphere, the solidity of which shall be equal to the superficies, what must be the diameter? Ans. 6.

3. If each side of a cube, which contains 64 inches, be 4 inches, what must be the side of a cube 27 times as large?

Solids are in proportion as the cubes of their similar dimensions.

The cube of 4 is 64; 27 times 64 is 1728; the cube root of 1728 is 12. Ans. 12 sq. inches.

4. What is the superficial and solid measure of a globe 15 inches in circumference?

Ans. Superficies, 71.62 square in.
Solidity, 113.989 cube in.

5. If an iron ball, 8 inches in diameter, weighs 72 pounds, what will be the weight of another, of the same metal, which is 4 inches in diameter?

Ans. 9 pounds.

6. If a silver sphere, 3 inches in diameter, be worth 25 dollars, what is the value of one which is 6 inches in diameter? The cube of 3 is 27; the cube of 6 is 216, which is 8 times 27. Ans. 200 dollars.

7. The wall of ancient Babylon was 60 miles in circumference, 87 feet thick, and 350 feet high. How many cubic feet did it contain?

Ans. 9646560000 cubic feet.

8. What are the solidities of a sphere, a cube, a cylinder, and a cone, the diameters and altitude of each being 10 inches?

The solidity of a sphere is equal to two-thirds, and a cone one third, of a cylinder of same base and altitude.

Ans. Cube 1000. cubic inches.
Cylinder, 785.4 " "
Sphere, 523.6 " "
Cone, 261.8 " "

9. If the diameter of the earth, at the equator, be 7920 miles, what is the circumference, area of the surface, and solidity of the earth?

Ans. Circumference, 24881. miles.
Area, 197061258 square miles.
Solidity, 260121860876 cubic miles.

10. What is the solidity of one of the frigid zones, allowing the diameter of the earth to be 7920 miles?

11. If the diameter of the earth be 7920 miles, and the diameter of the moon be 2160 miles, what is the difference of the areas of their surfaces?

Ans. 182403809280 square miles.

12. If the area of a circle be 16 inches, what is the area of the *curved surface* of a hemisphere of equal diameter?

13. If the area of the curved surface of a hemisphere be 45 inches, what is the area of the *entire surface* of a hemisphere of the same diameter?

14. If a hemisphere of gold be worth $500, what will be the value of the greatest cone which can be cut from it?

GEOMETRY.

PART FIFTH.

CHAPTER I.

MENSURATION OF ANGLES AND TRIANGLES.

SECTION I.

Definitions and Problems.

Every circle is divided into 360 equal parts, called degrees; each degree is divided into 60 equal parts, called minutes; each minute is divided into 60 equal parts, called seconds; seconds are divided in the same manner, and the parts are called thirds.

60 thirds are one second.
60 seconds are one minute.
60 minutes are one degree.
360 degrees are one circle.

A semicircle contains 180 *degrees; and a quadrant, or quarter of a circle, contains* 90 *degrees. A quadrant of a circle is formed by two radii perpendicular to each other, and, consequently, at a right angle; for all perpendicular lines form right angles. A fourth part of the circumference, then, is the measure of a right angle. Therefore, a* *right angle contains* 90 *degrees. An obtuse angle is greater than a right angle, and contains more than* 90 *degrees. An acute angle is less than a right angle, and contains less than* 90 *degrees.*

Obtuse and acute angles are called oblique angles.

It is often necessary, for practical purposes, to ascertain how many degrees an oblique angle contains; and also, to make an angle of any given number of degrees.

A convenient instrument for making and measuring angles is a protractor, which is a small graduated semi-circle, applied to the angle by placing the point marking the centre of the diameter at the vertex of the angle, and noting the number of degrees on the edge of the protractor.

A *quadrant* is an instrument for taking and measuring angles in a *vertical plane.*

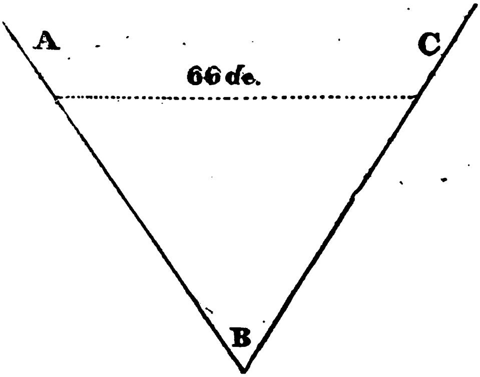
A *graphometer* is an instrument for taking and measuring angles in a *horizontal plane.*

PROBLEM I.

To find the number of degrees contained in any given angle.

From a scale of chords, with a pair of dividers, set off 60 *degrees from the vertex on each side of the angle. Then take the distance between the two points thus set off, and apply it to the same scale of chords, and it will give the number of degrees which the angle measures.*

1. How many degrees are contained in the angle A B C?



In the angle A B C, 60 degrees are set off from B to A, and from B to C. Then the distance from A to C is 66 degrees, the measure of the angle, A B C.

2. How many degrees are there in an angle which at 1 and $1\frac{1}{2}$ inch from the vertex opens 1 inch?

Ans. 39 degrees.

3. Draw four acute angles on the same side of a straight line, and measure them. If they are measured accurately the sum of them will be 180 degrees.

4. Draw one right angle and two acute angles on one side, and one obtuse and two acute angles on the other side of the same line. The sum of the six angles will be 360 degrees.

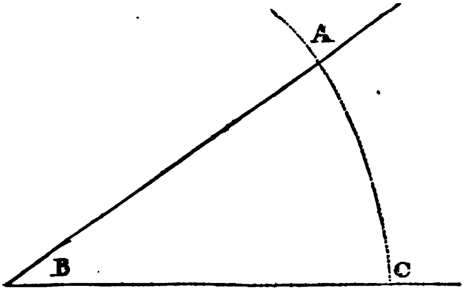
PROBLEM II.

To make an angle equal to any given number degrees.

Draw a right line of indefinite length. With the di-

viders extended 60 degrees, place one foot on the extremity of the line and describe an arc of a circle to intersect the right line. Set off the given number of degrees on the arc from the point of intersection. From the extremity of the right line, opposite the arc, draw a right line through the arc at the point, marking the given number of degrees; and the required angle will be formed.

1 To make an angle of 35 degrees.



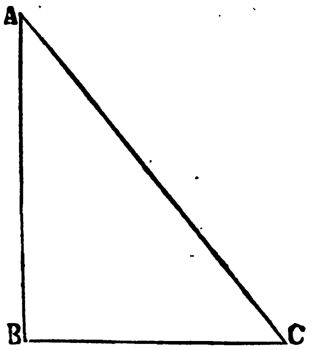
From B to C, set off 60 degrees; and describe the arc A C. From C. to A. on the arc, set off 35 degrees; then a line drawn from B through A forms the angle required.

2. What will be the length of a chord of an arc of 45 degrees drawn through an angle at the distance of three inches from the vertex? Ans. 2.3 inches.

3. Draw an angle of 15, and one of 40, one of 55, and one of 70 degrees, all on the same side of a straight line.

PROBLEM III.

Two sides of a right triangle being given, to find the length of the third side.



The line A B is the perpendicular.
The line B C is the base.
The line A C is the hypothenuse.

A. The base and perpendicular given to find the length of the hypothenuse.

From the sum of the squares of the base and perpendicular, extract the square root, the result will give the length of the hypothenuse.

1. If the base of a right triangle be 6, and the perpendicular 8, what will be the length of the hypothenuse?

$6\times8\times2+4=100$; the sum of the squares of the base and perpendicular. The square root of 100 is 10; therefore, the length of the hypothenuse is 10.

2. What is the length of the hypothenuse of a right triangle, the perpendicular of which is 6, and the base 4.5?
Ans. 7.5.

3. If the base of a right triangle be 87, and the perpendicular 116, what will be the length of the hypothenuse? Ans. 145.

B. The hypothenuse and base given, to find the length of the perpendicular.

From the difference of the squares of the hypothenuse and base, extract the square root; the result will give the length of the perpendicular.

1. If the hypothenuse of a right triangle be 10, and the base 6, what will be the length of the perpendicular?

$10+6\times4=64$, the difference of the squares of the hypothenuse and base; the square root of 64 is 8, therefore the length of the perpendicular is 8.

2. If the hypothenuse of a right triangle be 14, and the base $8\frac{2}{5}$, what is the length of the perpendicular?

Change the fraction to a decimal. ANS. 11.2.

3. What is the length of the perpendicular of a right triangle, the hypothenuse of which is 115, and base 69?

ANS. 92.

C. The hypothenuse and perpendicular given, to find the length of the base.

From the difference of the squares of the hypothenuse and perpendicular, extract the square root, the result will be the length of the base.

1. If the hypothenuse of a right triangle be 10, and the perpendicular 8, what will be the length of the base?

$10+8\times2=36$; the square root of 36 is 6; therefore, the length of the base is 6.

2. The hypothenuse of a right triangle is 25, and the perpendicular 20, what is the length of the base?

ANS. 15.

3. If the hypothenuse of a right triangle be $6\frac{2}{3}$, and the perpendicular $5\frac{1}{3}$, what is the length of the base?

$6\frac{2}{3}+5\frac{1}{3}=12\times1\frac{1}{3}=16$, the difference of the squares of the hypothenuse and perpendicular; the square root of 16 is $4=$ the length of the base.

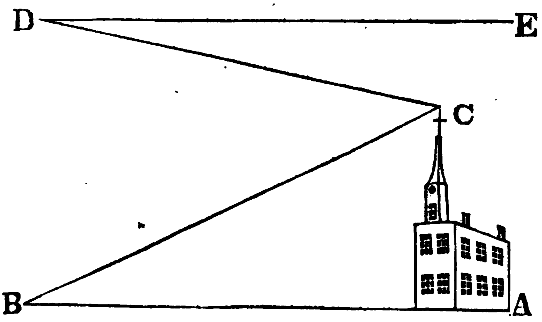
SECTION II.

Horizontal, Vertical, and Inclined Planes.

A smooth sheet of paper lying flat upon a level table represents a *horizontal plane;* a smooth sheet of paper placed edgewise on the table, and at right angles with it, represents a *vertical plane*; and if placed edgewise on the table, and forming oblique angles with it, represents an *inclined plane.*

Angles of Elevation and Depression.

When angles are taken in a vertical plane, the angle formed by a horizontal line, at the extremity of which the eye of the observer is placed, and a line drawn to an object above it is called an angle of elevation. An angle is formed by a horizontal line, and a line drawn to an object below it, is called an angle of depression.



The angle A B C is the angle of elevation of the object at C, taken from the point B. And the angle C D E, is the angle of depression of the object at C, taken from the point D.

The preceding principles are of great practical utility in the mensuration of heights and distances; and ought to be attentively studied and well understood, before attempting the solution of the following Problems.

CHAPTER II.

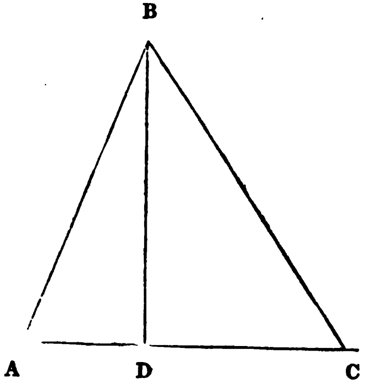
MENSURATION OF HEIGHTS AND DISTANCES.

PROBLEM I.

To find the distance of an object from the place of observation on a horizontal plane.

Take the bearings of the object from two points of observation. Draw a horizontal line, and from a scale of equal parts, set off the distance between the two points; make the angles equal to the bearings of the object and the point of intersection opposite the right line will mark the position, and the length of the lines will give the distance of the object.

1. It is required to find the distance of the object at B, from the points of observation A and C, and also the perpendicular distance, from a line drawn between them.



The bearing of the object B from the point A, is 79 degrees, and from the point C, it is 55 degrees. The distance from A to C is set off 167 feet. The point of intersection of A B and B C marks the position of the object. And the distance from A to B measured from the same scale from which the distance from A C was taken, is 176 feet, and from C to B, the distance is 196 feet, and the perpendicular distance from D to B is 164 feet.

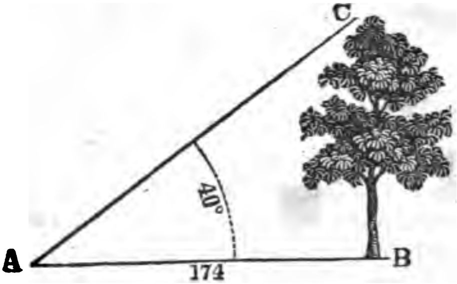
2. At the two extremities of a wharf, 250 feet wide, the bearings of a ship are ascertained to be 65 and 75 degress ; at what distance is the ship ?

PROBLEM II.

To find the height of an object standing on a horizontal plane.

From a scale of equal parts, set off the horizontal distance of the object from the place of observation, then take the angle of elevation, and the points of intersection with the object marks the height of the object and the distance from the place of observation.

1. It is required to find the height of a tree at 174 feet distant from the place of observation, allowing the height of the instrument to be five feet, and diameter of the tree four feet.



Let A B represent the distance, 100 feet. At B erect a line perpendicular to A B. The angle of elvation of the top of the tree is found to be 45 degrees. Draw the arc and set off 45 degrees. Then the line A C passing through the extremity of the arc, marks the height of the tree at the point of intersection with B C. Measure B C from the same scale of equal parts, to which add five feet for the height of the instrument, gives the height of the tree 134 feet.

2. At 300 feet distance from the bottom of a fort standing on a horizontal plane, the angle of elevation of the top of a tower is found to be 40 degrees; the height of the instrument, while in use, being 5 feet 6 inches. What is the height of the tower?

The following question may be solved by the rule for finding the perpendicular of a right triangle, the other sides being known.

3. The distance to the top of a tree from the place of observation is known to be 200 feet; and to the trunk of the tree, the diameter of which is 10 feet, the distance is 115 feet. What is the height of the tree?

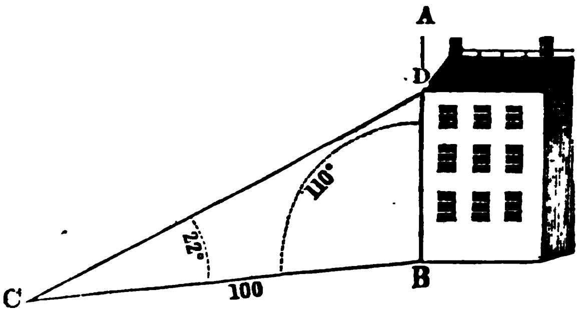
ANS. 160 feet.

PROBLEM III.

To find the height of an object standing on an inclined plane.

With the quadrant, take the angle which the object makes with the downward slope, and at a given distance, take the angle which it makes with the upward slope; draw the angles, and they will give the slope of the hill, and the height of the object.

1. A house standing on the declivity of a hill, makes an angle of 110 degrees with the downard slope, and at the distance of 100 feet down the hill, the angle of elevation of the top of the house is 15 degrees with the upward slope. Required the height of the house.



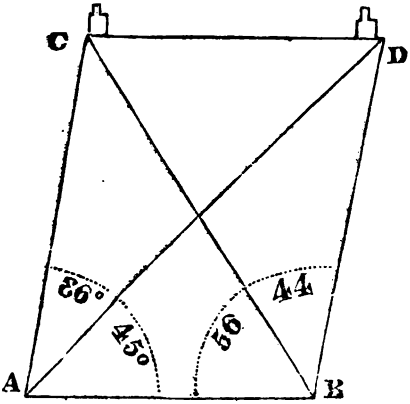
Draw the vertical line A B indefinitely, to represent the side of the house. At B make an angle with the right line of 110 degrees, and set off B C, passing through the extremity of the angle equal to 100 feet; this gives the slope of the hill. At C, make an angle of 15 degrees, and draw C D through the angle; and the intersection of C D with A B, marks the top of the building; measure B D, and it will give the height of the house.

PROBLEM IV.

To find the distance between two objects situated in a horizontal plane.

From two points of observation take the bearings of the two objects. From a scale of equal parts set off the distance between the two places of observation; draw the angles, and the points of intersection opposite mark the positions of the objects, and the lines drawn between the points are the measure of the distances.

It is required to find the distance between two objects, C and D, on a horizontal plane.



Let A and B represent the two places of observation, 200 yards apart. From the point A, the bearings of the objects are found to be 45 degrees and 81 degrees; and from the point B the bearings are 56 degrees and 100 degrees. Draw the angles D A B and C A B, and also the angles C B A and D B A; the points of intersection at C and D mark the positions, and the line C D gives the distance between the two objects.

The lines A C and A D give the distances of the objects from the point A, and the lines B C, B D give their distances from the point B.

A B = 200. A D = 346. A C = 242. B C = 304. B D = 254, and C D = 209.

2. From two points of observation, 400 feet apart, the bearings of two objects are found to be, from one point of

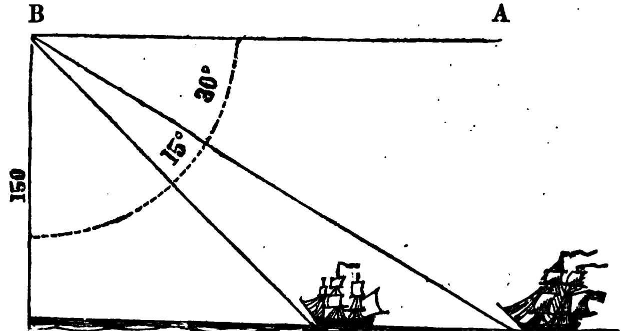
observation, 35 degrees and 60 degrees; and from the other, 47 degrees and 85 degrees. What is the distance between the two objects?

PROBLEM V.

To find the horizontal distance between two objects in the same vertical plane, the bearings taken from an elevated point of observation.

Take the angles of depression of the two objects, draw a vertical and a horizontal line to meet, set off on the verical line the height of the place of elevation, from which draw the angles of depression; and the points of intersection, with the horizontal line, mark the positions of the objects. From a scale of equal parts, measure on the horizontal line the distance between the objects.

1. From the top a hill, 150 feet above the level of the sea, two vessels are observed; the angle of depression of one is 45 degrees; that of the other, more remote, is 30 degrees. What is the distance between them?



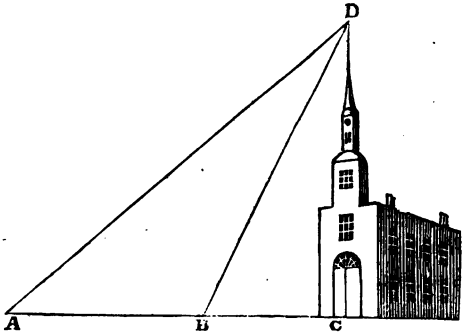
Let B C equal 150 feet, the altitude of the place of observation; draw C E perpendicular to B C, and B A parallel to C E, for the purpose of making the angles of depression. Set off the angle A B D equal to 45 degrees; and the angle A B E equal to thirty degrees. The intersections at D and E determine the positions of the vessels. Measure D to E, and it will give the distance between the vessels.

PROBLEM VI.

To find the height and distance of an inaccessible object, standing on a horizontal plane.

Take the angles of elevation of the object at two accessible points of observation in the same vertical plane; draw the angles, and the points of intersection will mark the position, and the measure of the lines will give the height and distance of the object.

1. The angle of elevation of the top of a steeple on the opposite side of a river is found to be 66 degrees, and 125 feet further distant in the same vertical plane, the angle of elevation is found to be 42 degrees. What is the height of the steeple, and the distance of its base from the first place of observation?



Draw A C indefinitely, and set off from A to B 125 feet. At A, make an angle of 42 degrees, and draw the line A D. At B, make an angle of 66 degrees, and the intersection of B D with A D, determines the height of the steeple. Draw the perpendicular, C D, and its intersection with A C marks the position of its base; measure from B to C, and it will give the distance from the first place of observation; to which, add the distance from B to A, and it gives the distance from the second place of observation.

2. Two persons, 175 feet apart, on the same level, and in the same vertical plane, find the angles of elevation of the top of a tower standing on the opposite bank of a river, to be 39 and 25 degrees. What is the height of the tower, and its distance from each place of observation?

3. The altitude of a steeple, by observations from the centre of a village, is 28 degrees; and advancing 30 yards nearer in the same vertical plane, the altitude is found to be 45 degrees. What is the height of the steeple, and distance of its base from the last place of observation?

PROBLEM VII.

To find the height of an object by knowing the greatest distance at which it is visible.

To the square of the distance of the object, add one fourth of the square of the diameter of the earth; from the square root of this sum, subtract the radius of the earth, the remainder is the height of the object.

1. What is the height of a mountain, the summit of which is visible 40 miles at sea? ANS. 1056 feet.

2. If the top of a lighthouse can be seen 20 miles at sea, what is its height from the surface of the sea ?
Ans. 266 feet.

PROBLEM VIII.

To find the greatest distance at which an object is visible, the height of which is known.

Multiply the diameter of the earth by the height of the object ; to the product, add the square of the height ; the square root of this sum will give the distance required.

1. If a mountain be one fifth of a mile high, at what distance is its summit visible ? Ans. 39$\frac{4}{5}$ miles.

2. How far can a mountain, 2640 feet high, be seen ?
Ans. 63 miles.

3. At what distance is a mountain, 2 miles high, visible ? Ans. 126 miles.

PROLEM IX.

To find the diameter of the earth, by knowing the height of a mountain, and the distance at which it is visible.

Divide the square of the distance by the height of the object ; and from the quotient subtract the height ; the remainder will give the diameter of the earth.

1. If the height of a mountain be 2640 feet, and its summit can be seen 62.93 miles, what is the diameter of the earth ? Ans. 7922 miles.

If a mountain, one mile high, be visible 89 miles, what is the diameter of the earth ?
Ans. 7921 miles.

CHAPTER III.

MISCELLANEOUS EXAMPLES.

The following questions may be solved by the rules for finding the base, perpendicular and hypothenuse of a right triangle.

1. From the top of a tower, 180 feet high, standing on the opposite bank of a river, the distance is 225 feet, what is the width of the river? Ans. 8 rods, 1 yard.

2. What is the height of a house which requires a ladder 52 feet long, standing 20 feet from the house, to reach the top? Ans. 48 feet.

3. A fort on the bank of a river 225 feet wide, is surmounted by a wall 74.5 feet high. What is the distance from the opposite bank to the top of the wall? Ans. 237 feet.

4. A. travels south 105 miles; and B, from the same point, travels west 140 miles. How far apart are they? Ans. 175 miles.

5. The height of a roof is 7½ feet, and the span 20 feet. What is the length of the rafters?

The span is the breadth of the building. Ans. 12½ feet.

6. How long must a ladder be to reach the roof of a house, 50 feet high, the ladder standing 20 feet from the house? Ans. 54 feet.

7. It is required to prop the wall of a building with a timber, 39 feet long; the upper end of the timber to be placed against the wall 36 feet from the ground. How far from the side of the building must the other end be placed? Ans. 15 feet.

8. A fortified town is surrounded by a wall 32 feet high, and outside of the wall, by a ditch, 24 feet broad. What must be the length of a scaling-ladder with which to mount the wall? Ans. 40 feet.

9. Two vessels depart from the same port at the same time; one sails 9 miles an hour in a direction due east; the other sails 12 miles an hour due south. How far apart will they be at the end of ten hours?

Ans. 150 miles.

10. Two ships leave port at the same time; one sails due east $5\frac{1}{4}$ miles in an hour; the other due north 7 miles an hour. At the end of 12 hours, they cast anchor, and despatch a boat from each vessel to the other. How long will it take for the boats to meet, rowing at the rate of 5 miles an hour? and in what direction must the boats go?

Ans. The boats will meet in 10 hours and 30 minutes. One must go north-west, the other south-east.

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